Lifting Synchronization Barriers

ECE 826

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Today

- Stragglers in Synchronous Distributed Optimization

- Lifting Synchronization Barriers

- HogWild!

Stochastic Gradient Descent

$$\min_{\mathbf{w}} \frac{1}{n} \sum_{i=1}^{n} \ell(\mathbf{w}; \mathbf{z}_i)$$

loss for data point i

- **Idea** ('50s, '60s [Robbins, Monro], [Widrow, Hoff]):

  Sample a data point + locally optimize.

SGD: *An Über-algorithm*

$$\mathbf{w}_{k+1} = \mathbf{w}_k - \gamma \cdot \nabla \ell(\mathbf{w}_k; \mathbf{z}_{i_k})$$

Stochastic Gradient Descent

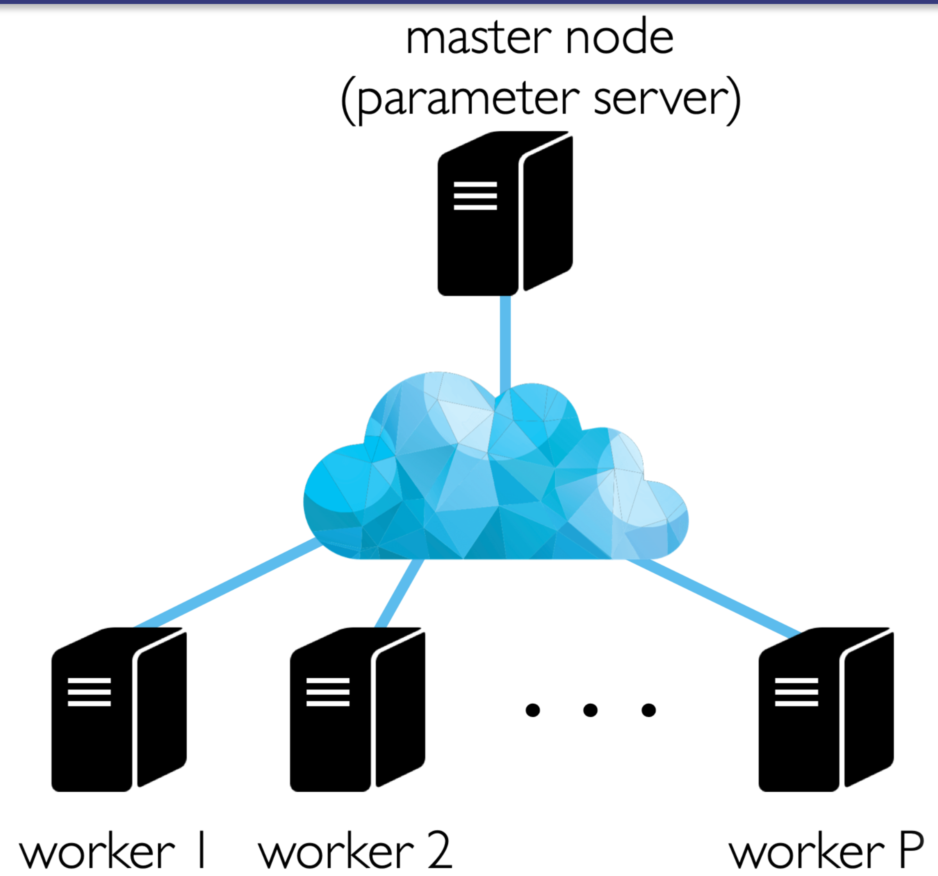
SGD can take years on large nlp models even on a single high end GPU

Goal:

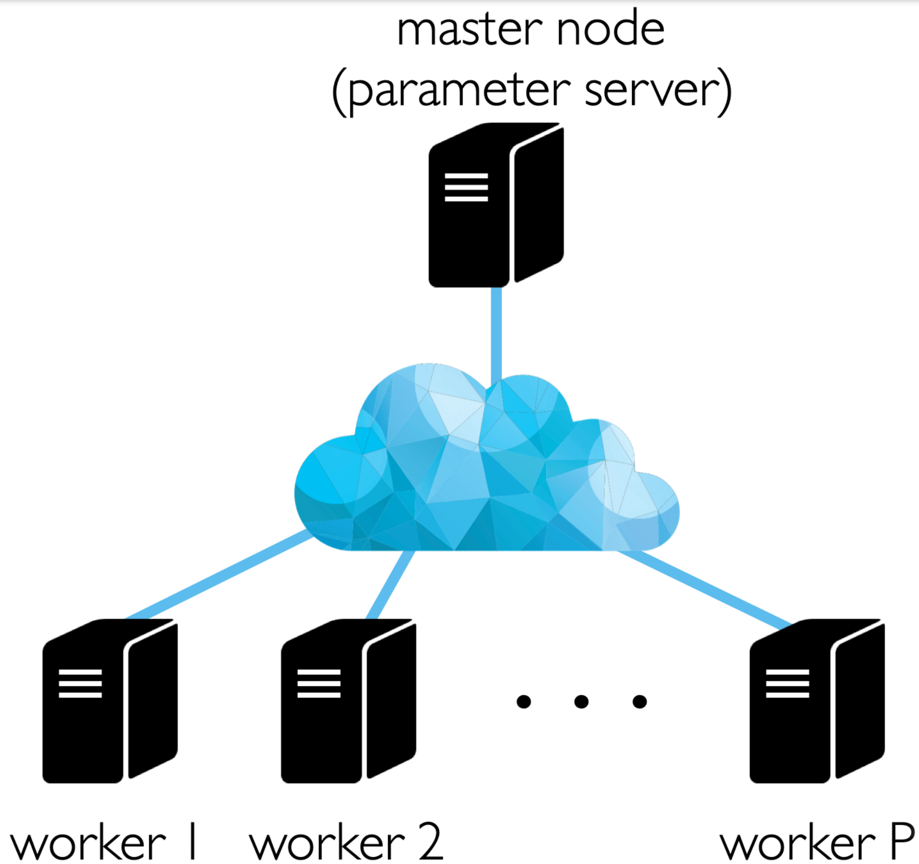
Speed up Machine Learning

Scaling Up SGD

Synchronous computation



Algorithm of choice: minibatch SGD



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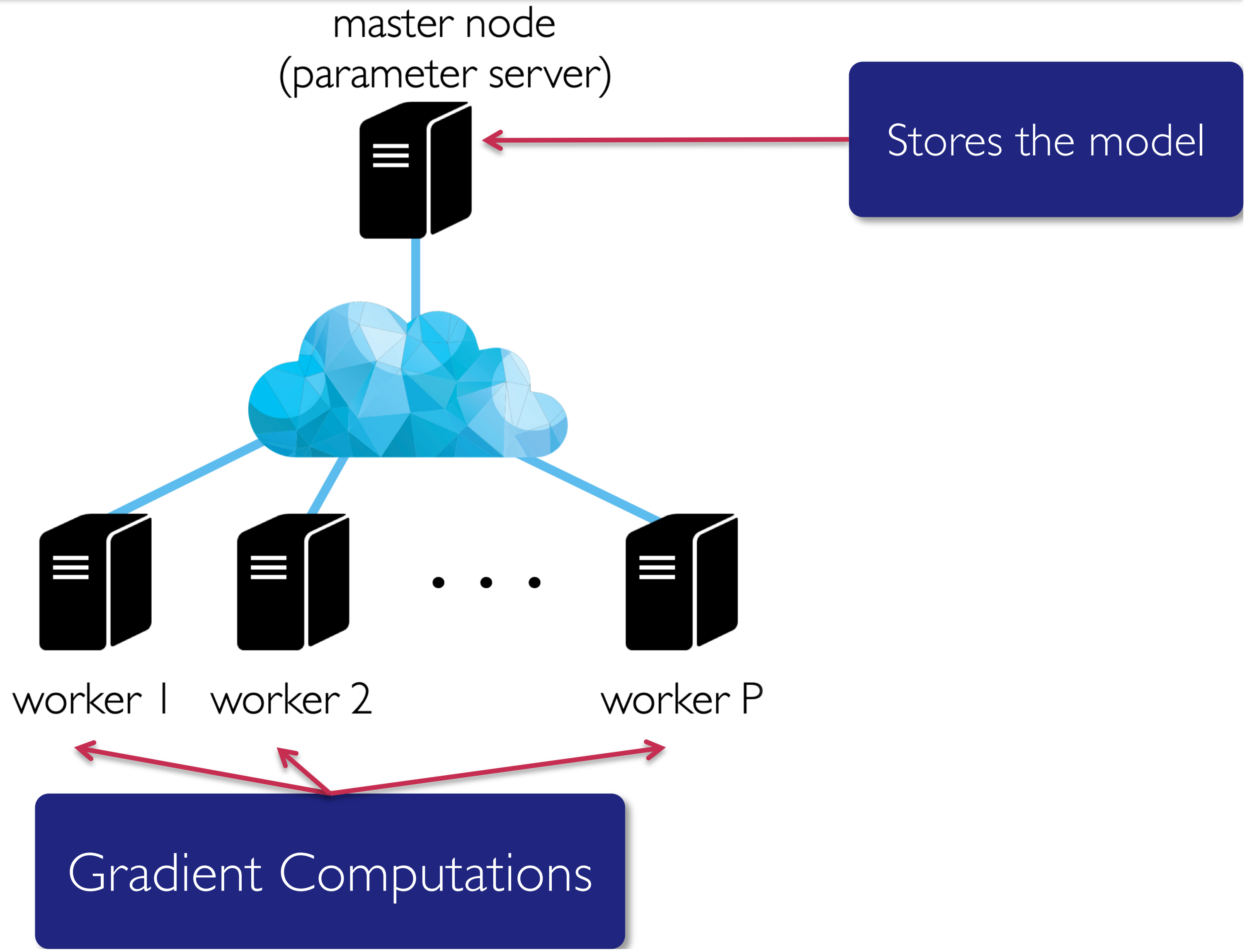

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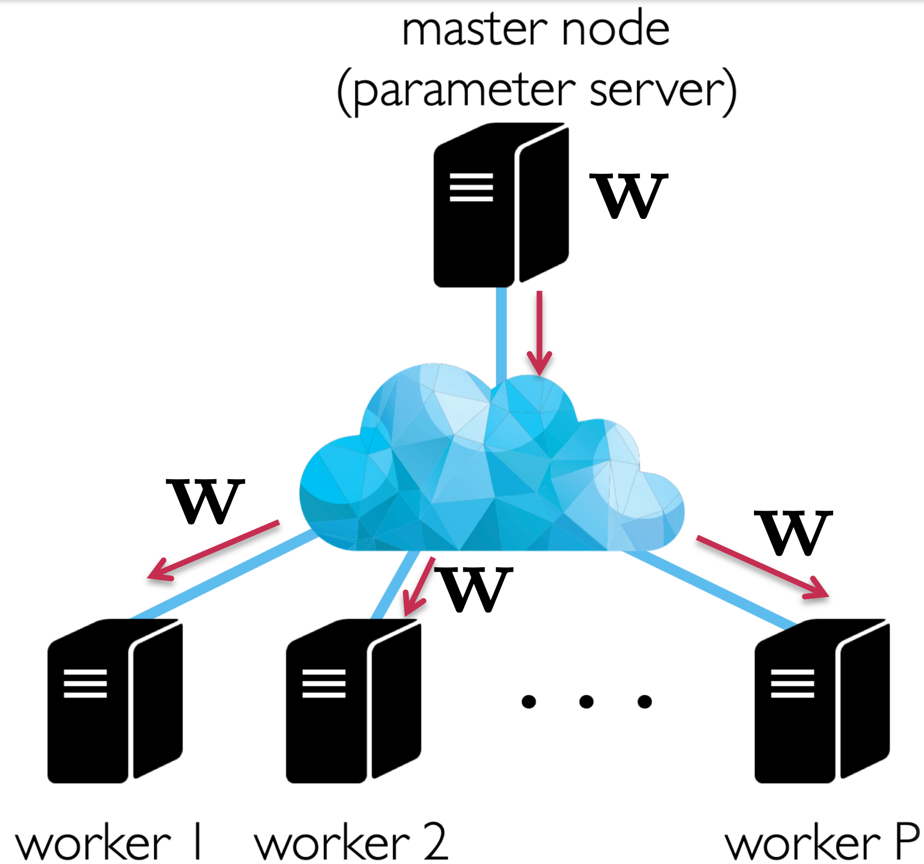

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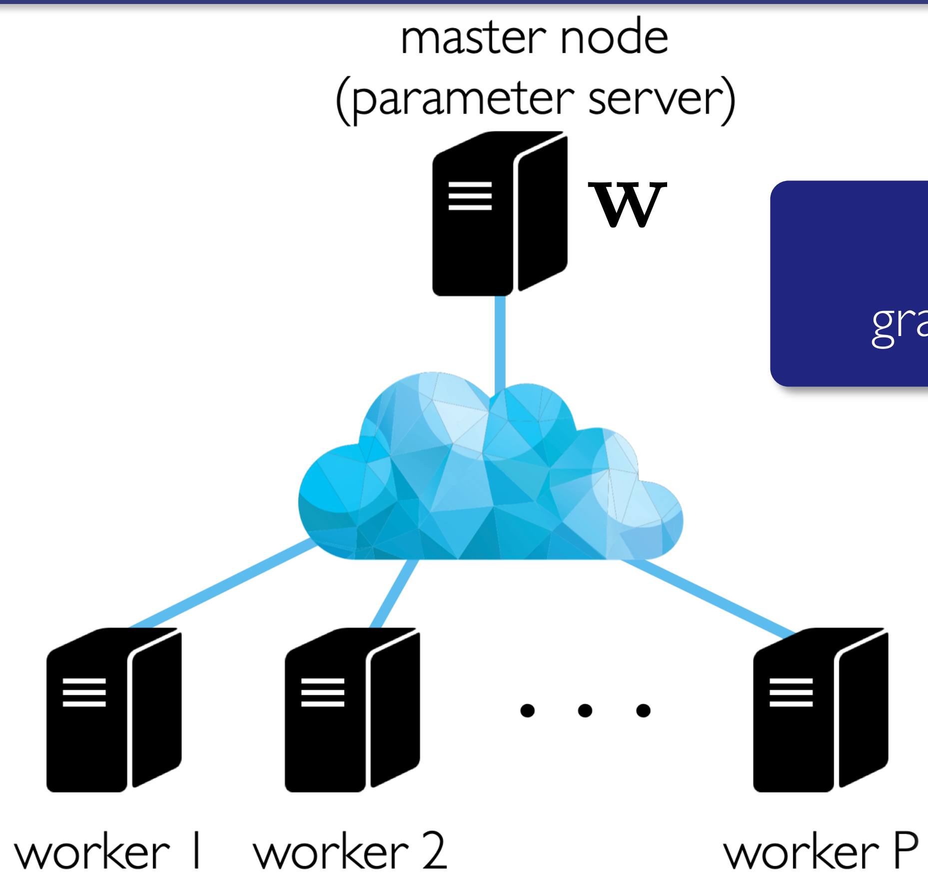

B = batch size,
gradients / iteration

$$\mathbf{g}_1 = \sum_{i \in \mathcal{S}_1} \nabla \ell(\,(\mathbf{x}_i, y_i); \mathbf{w}\,)$$

$$\mathbf{g}_P = \sum_{i \in \mathcal{S}_P} \nabla \ell(\,(\mathbf{x}_i, y_i); \mathbf{w}\,)$$

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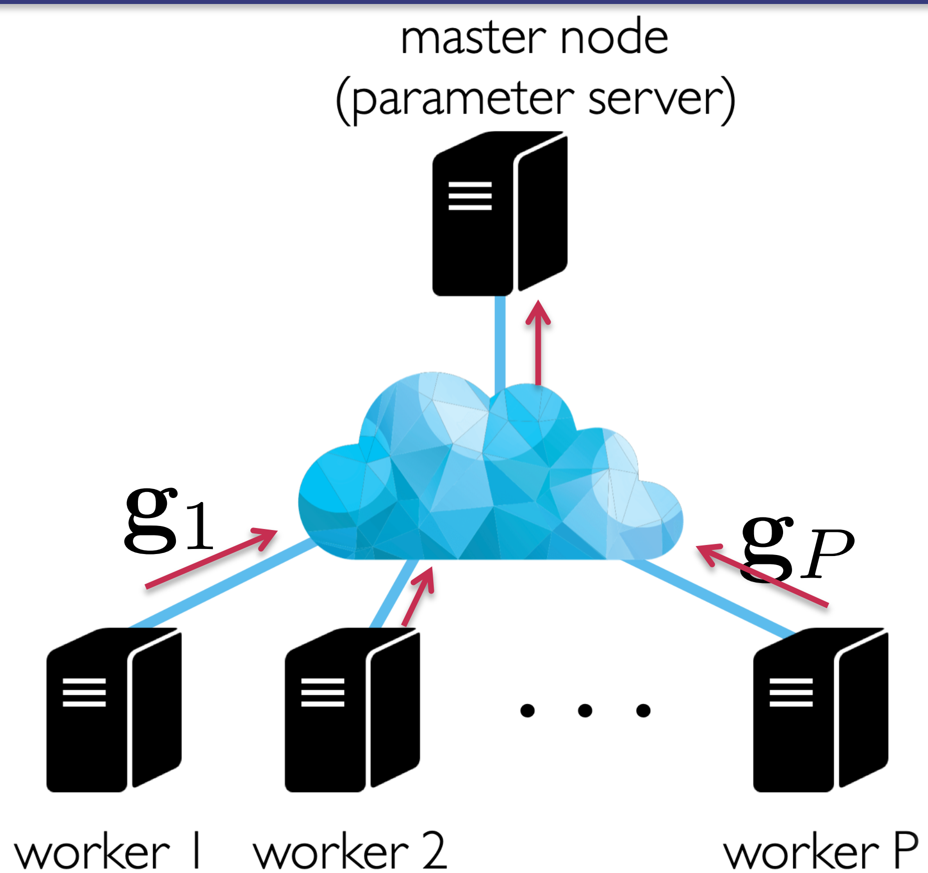

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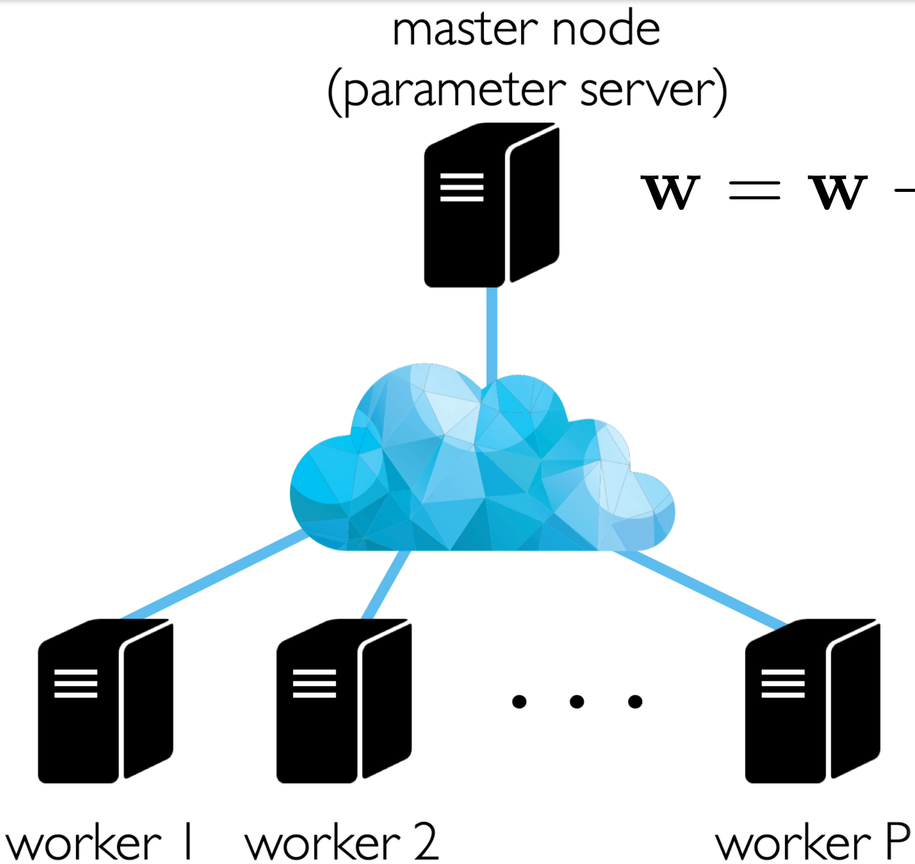
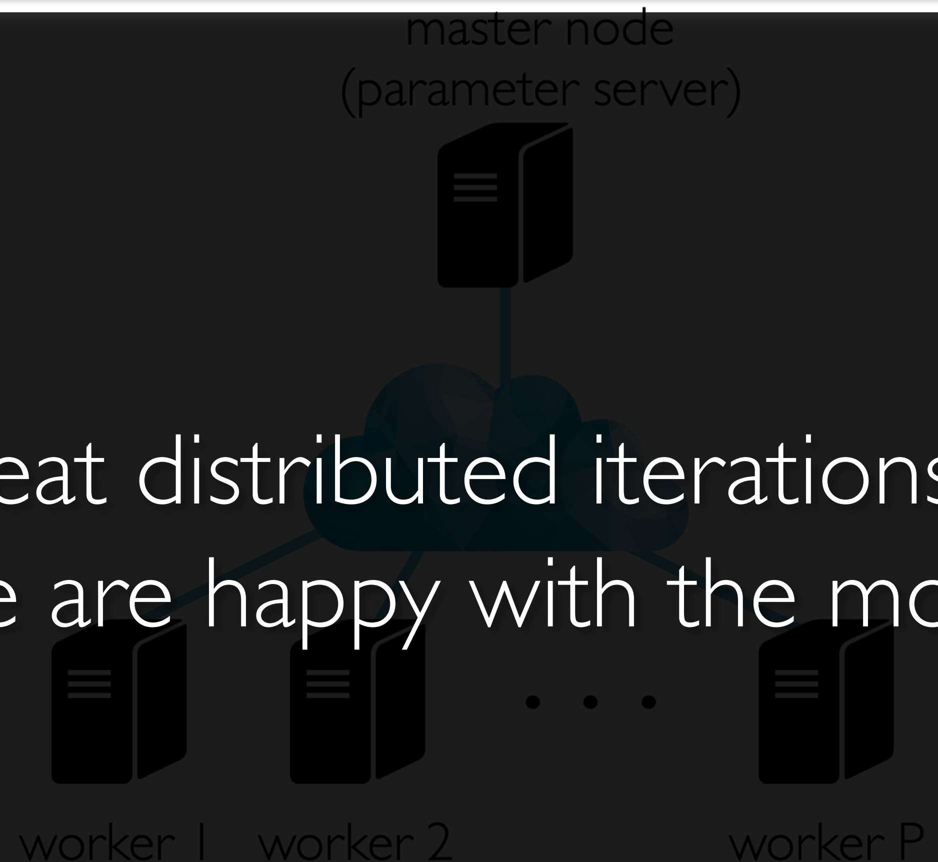


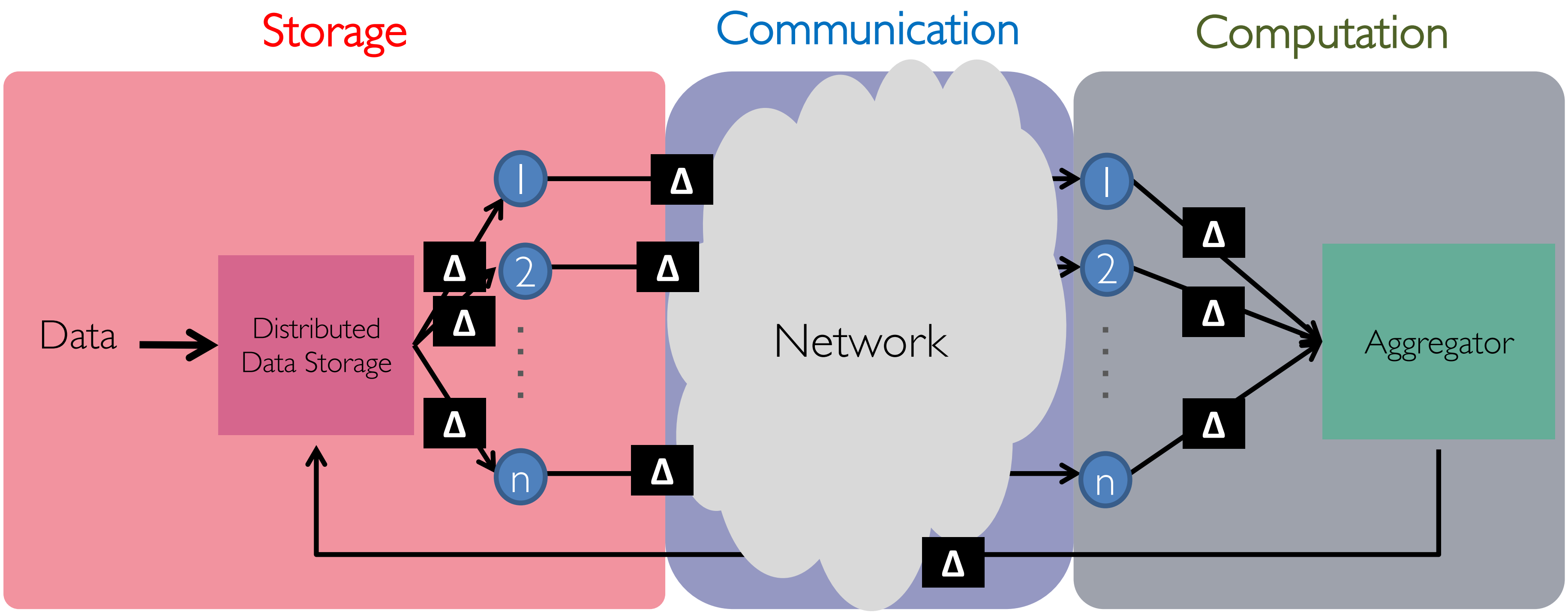
$$\mathbf{w} = \mathbf{w} - \gamma \sum_{i=1}^{P} \mathbf{g}_i$$

Algorithm of choice: minibatch SGD



Repeat distributed iterations until we are happy with the model

Large-scale Distributed Machine Learning Systems



"The scale and complexity of modern Web services make it ***infeasible*** *to eliminate all latency variability."*

Jeff Dean, Google.

Stragglers

- Ideal compute time per node ~ O(total_time/P)
- But there is a lot of randomness:
  - Network/Comm Delays
  - Node/HW Failures
  - Resource Sharing

- What if time per node is a random variable:

  $$X = \text{constant} + \text{Exp}(\lambda)$$

Lemma:

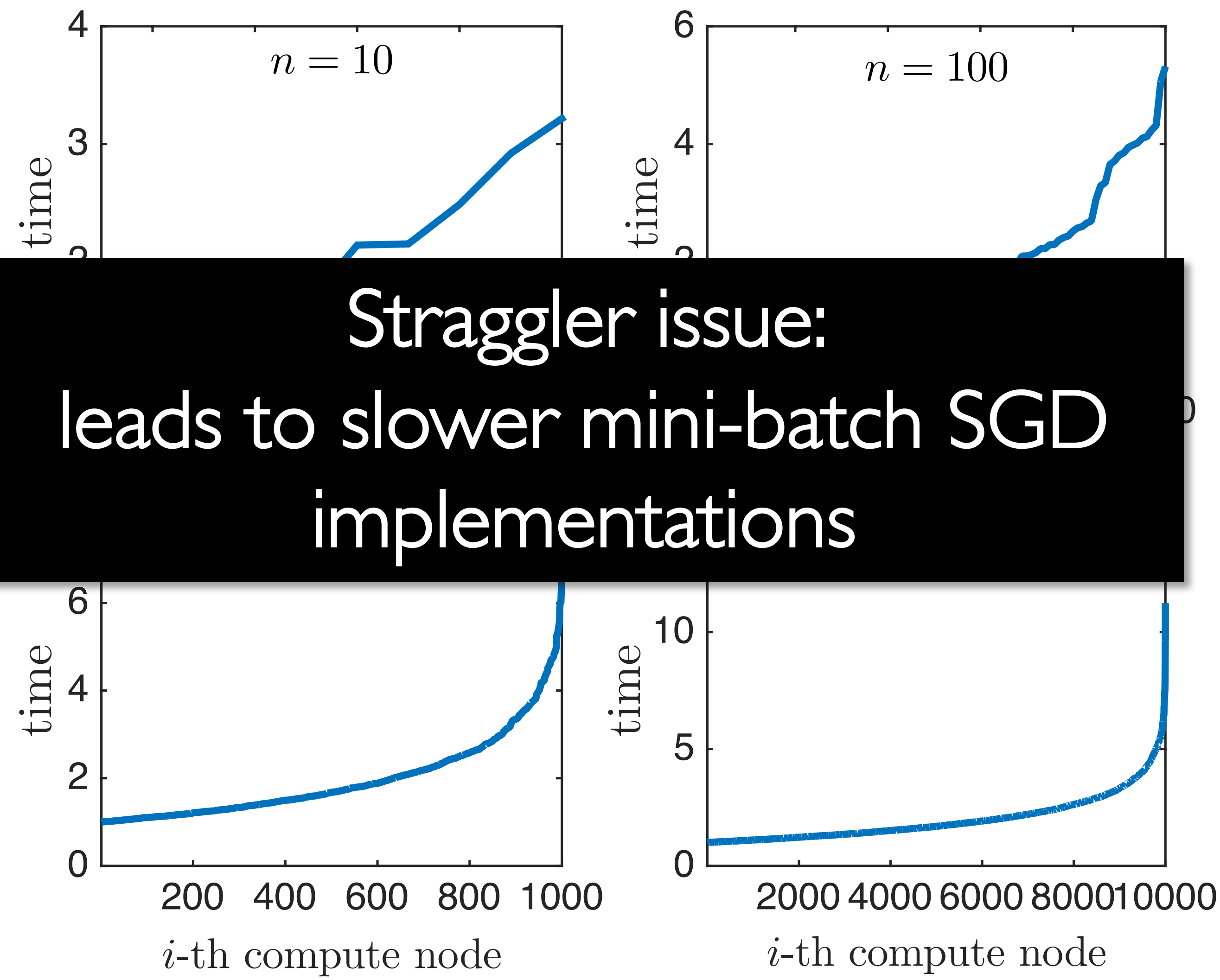
$$\mathbb{E}\{X_{(i)}\} = 1 + \frac{1}{\lambda} \sum_{n-i+1}^{n} \frac{1}{i}$$

Remark:

Slowest node is log(n) times slower than fastest

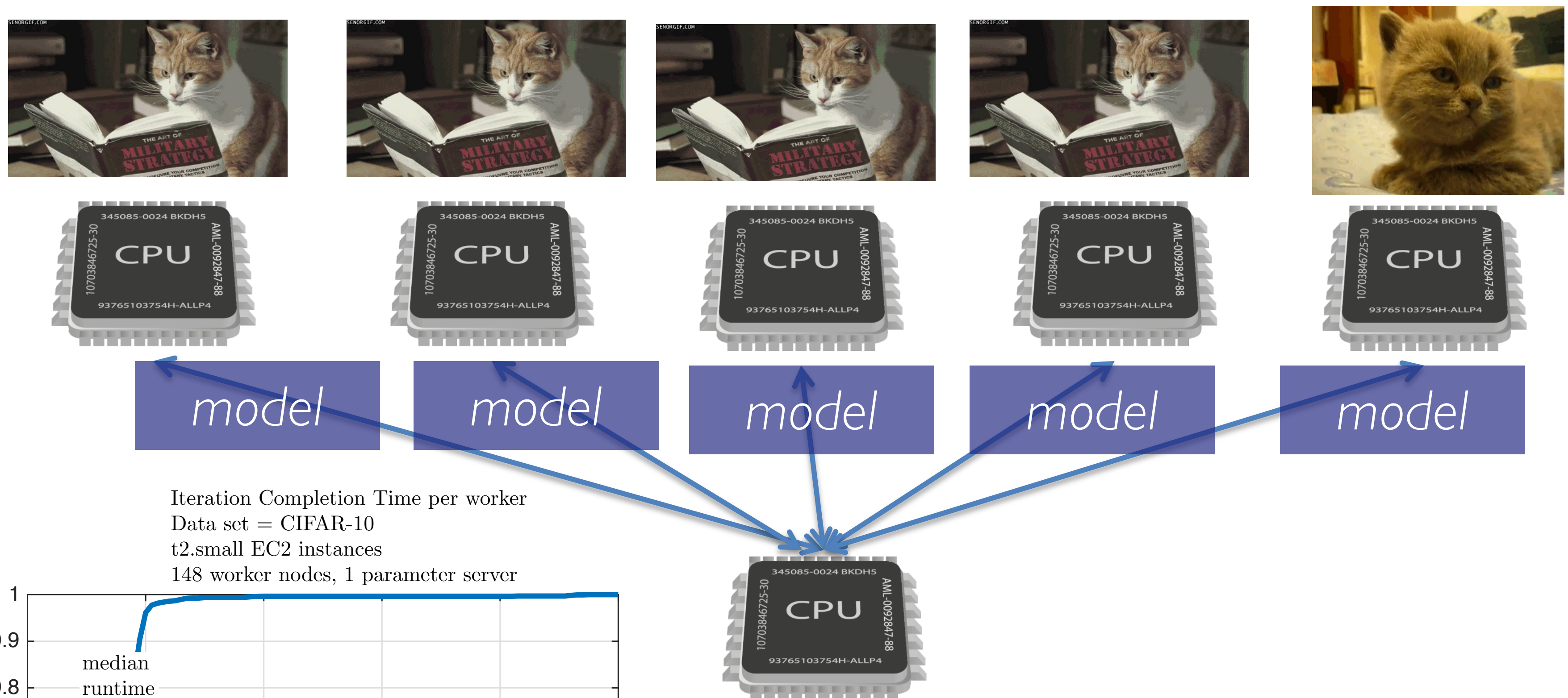
Simulation

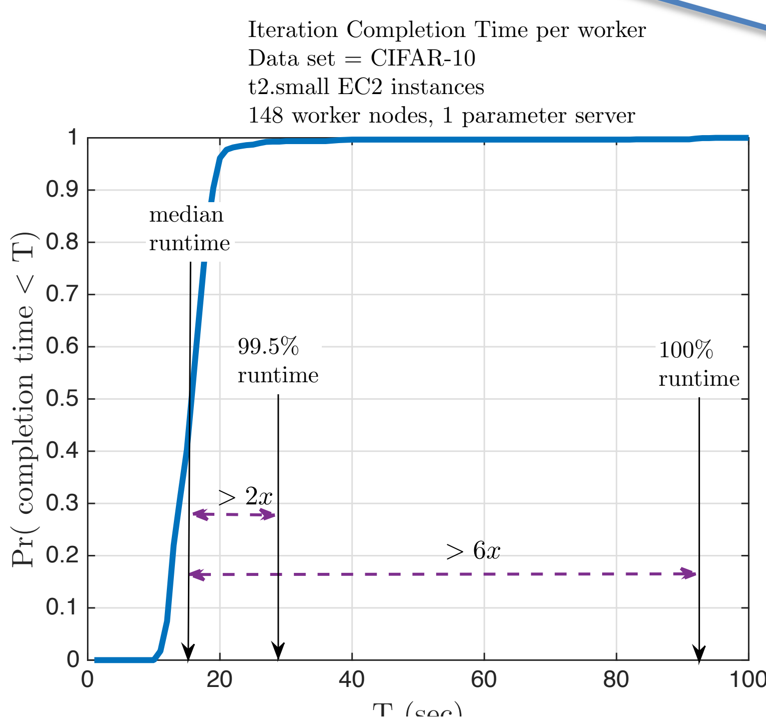
- X(t) = 1+ Exp(0.5), n = 10, 100, 1000, 1000



Straggler issue:
leads to slower mini-batch SGD implementations

Bottleneck: Straggling Learners

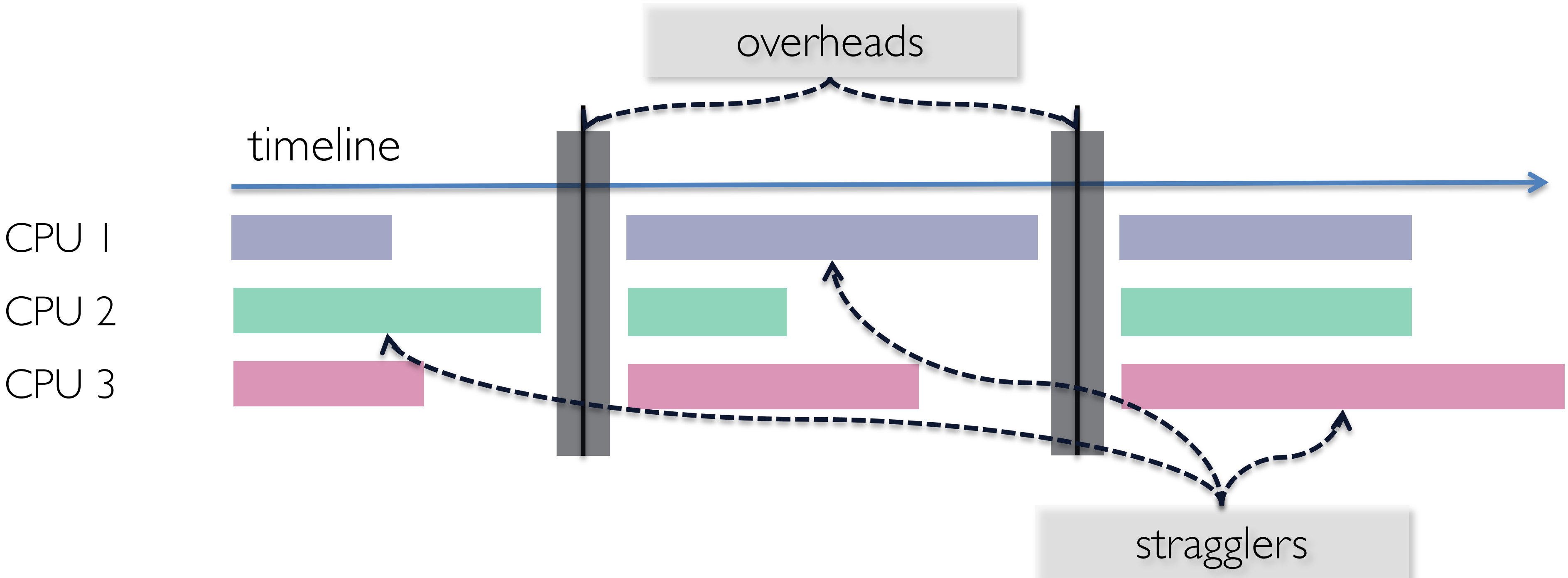




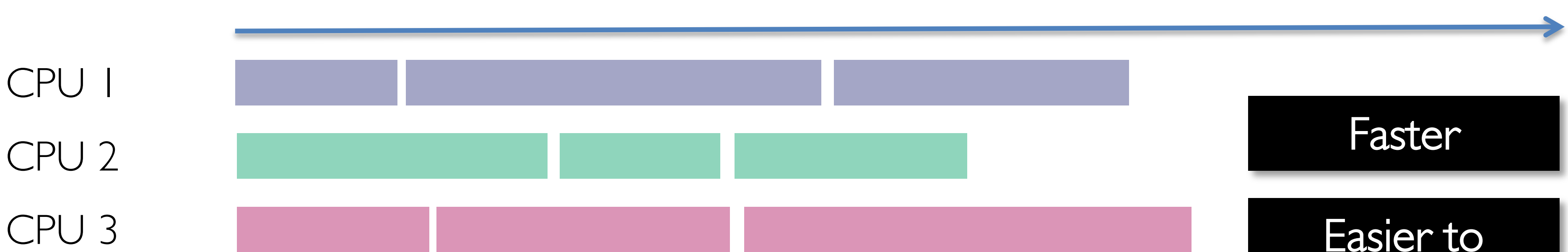
Can we "robustify" distributed ML against stragglers?

Measured on Amazon AWS

A case against Synchronization



Asynchronous World



Faster

Easier to Implement

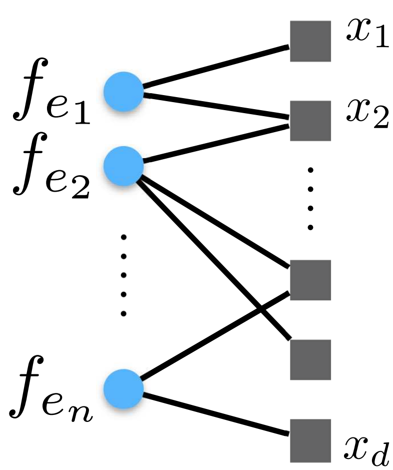
Asynchronous SGD on Sparse Functions

SGD on sparse functions

$$f(x) = \sum_{e \in \mathcal{E}} f_e(x_e)$$

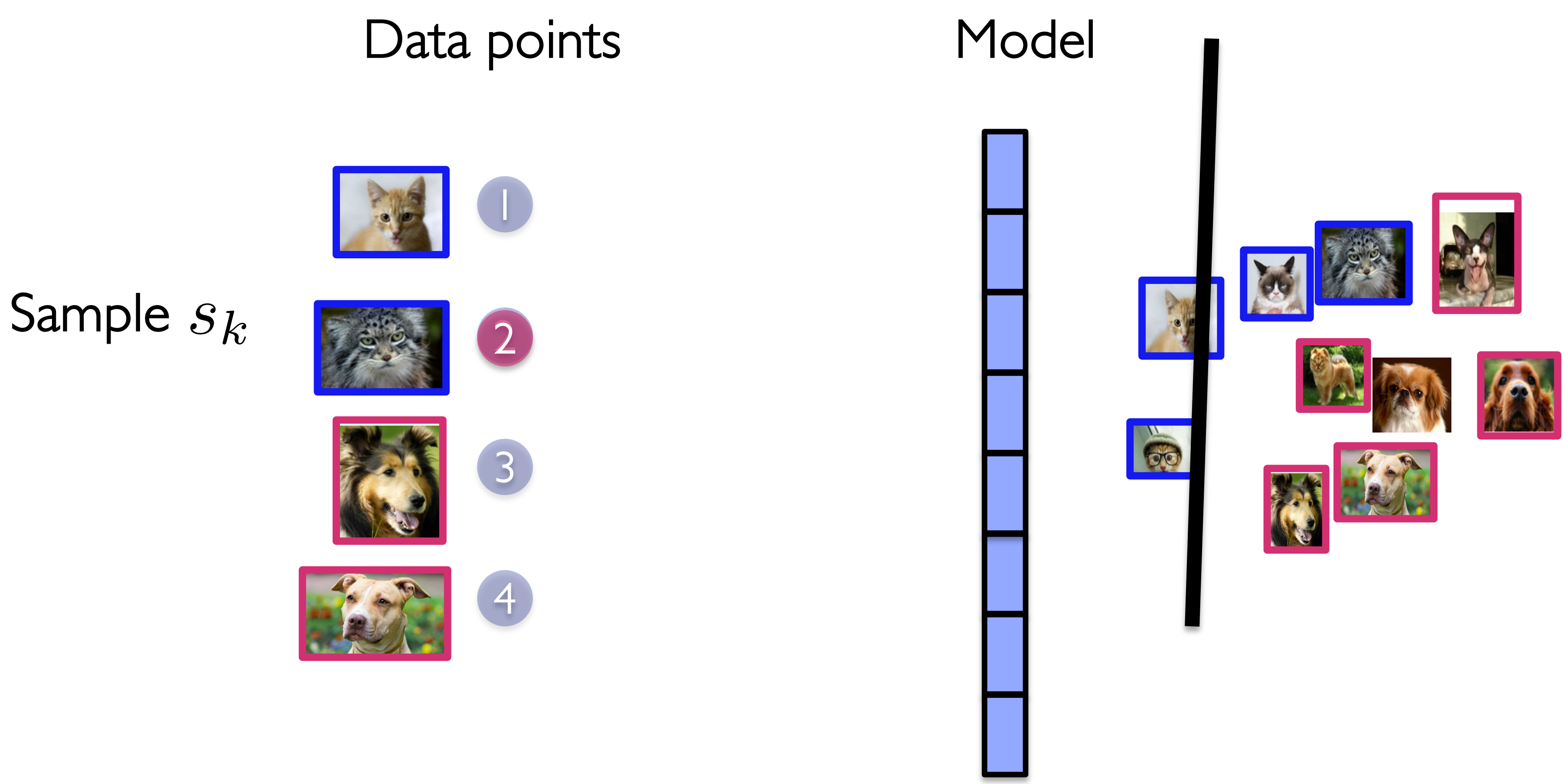
- Def:
  Hyperedge e = the subset of variables that f_e depends on

- The function-variable graph



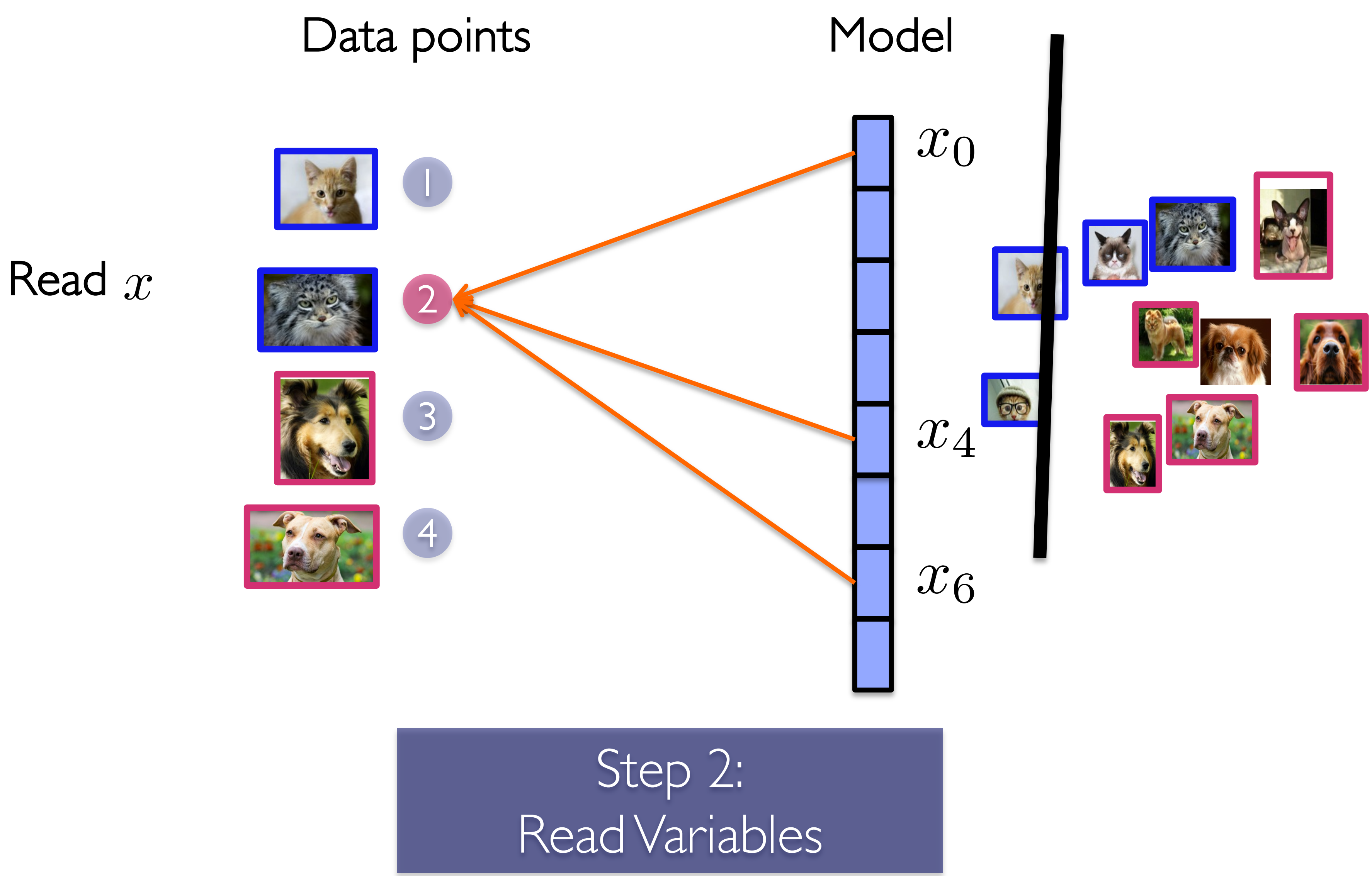
Matrix Fact./Comp.
Graph cuts
Graph/text Classification
Topic Modeling
Dropout
…

SGD on sparse functions



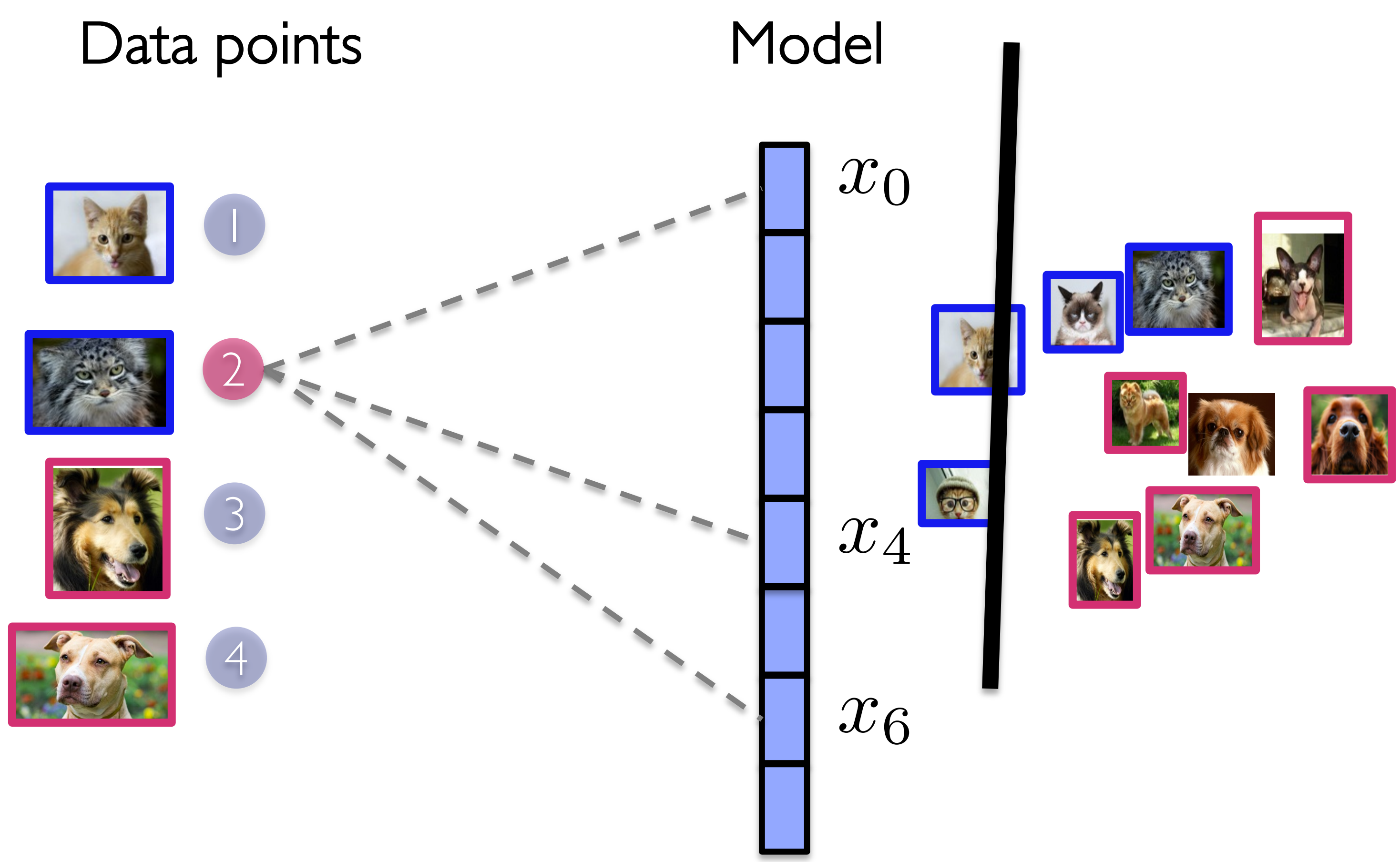
Step 1:
Pick random data point

SGD on sparse functions



SGD on sparse functions

Compute grad.
of local loss
$\nabla f_{s_k}(x)$

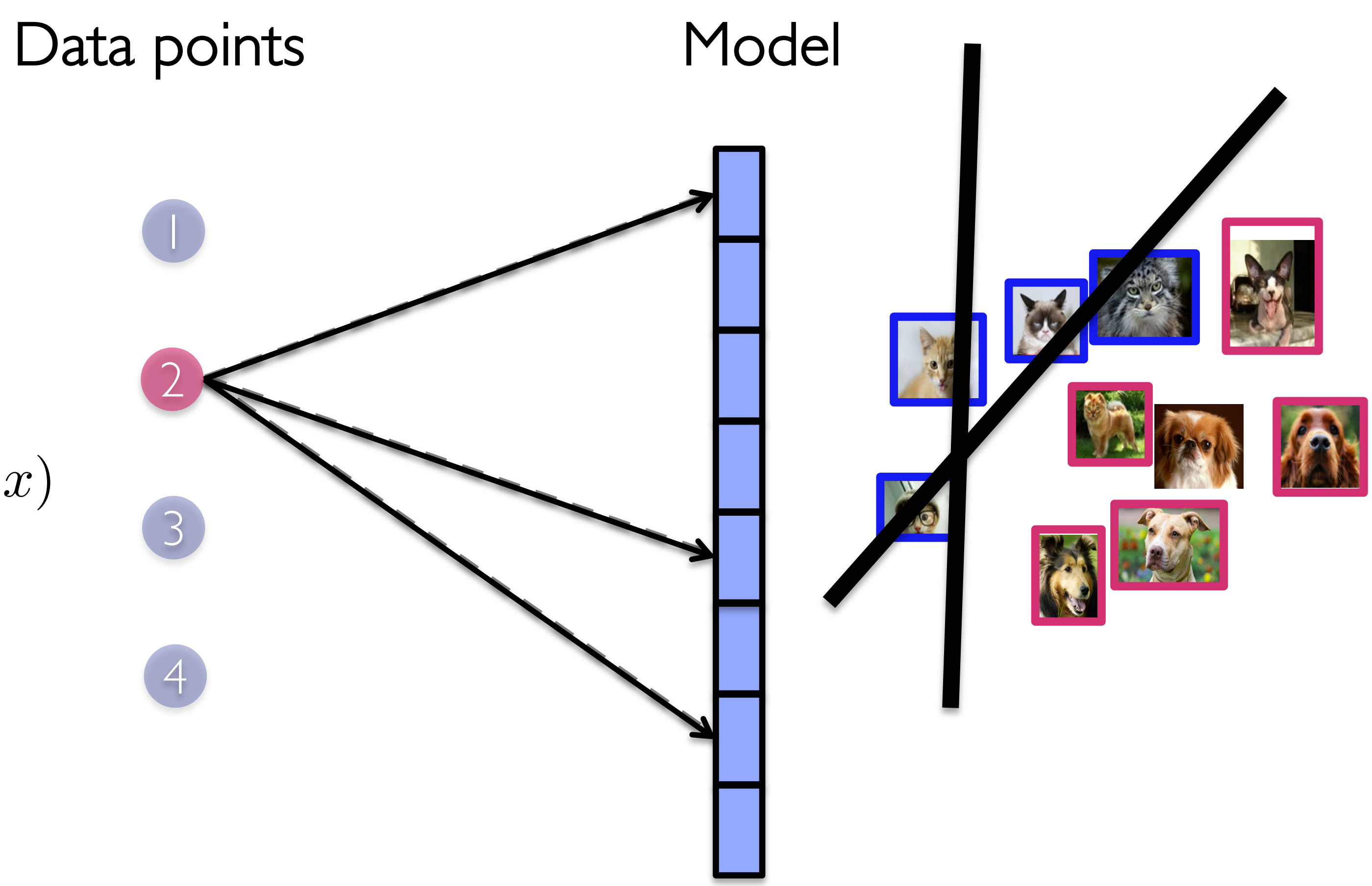


Step3:
Compute local function

SGD on sparse functions

Update Model

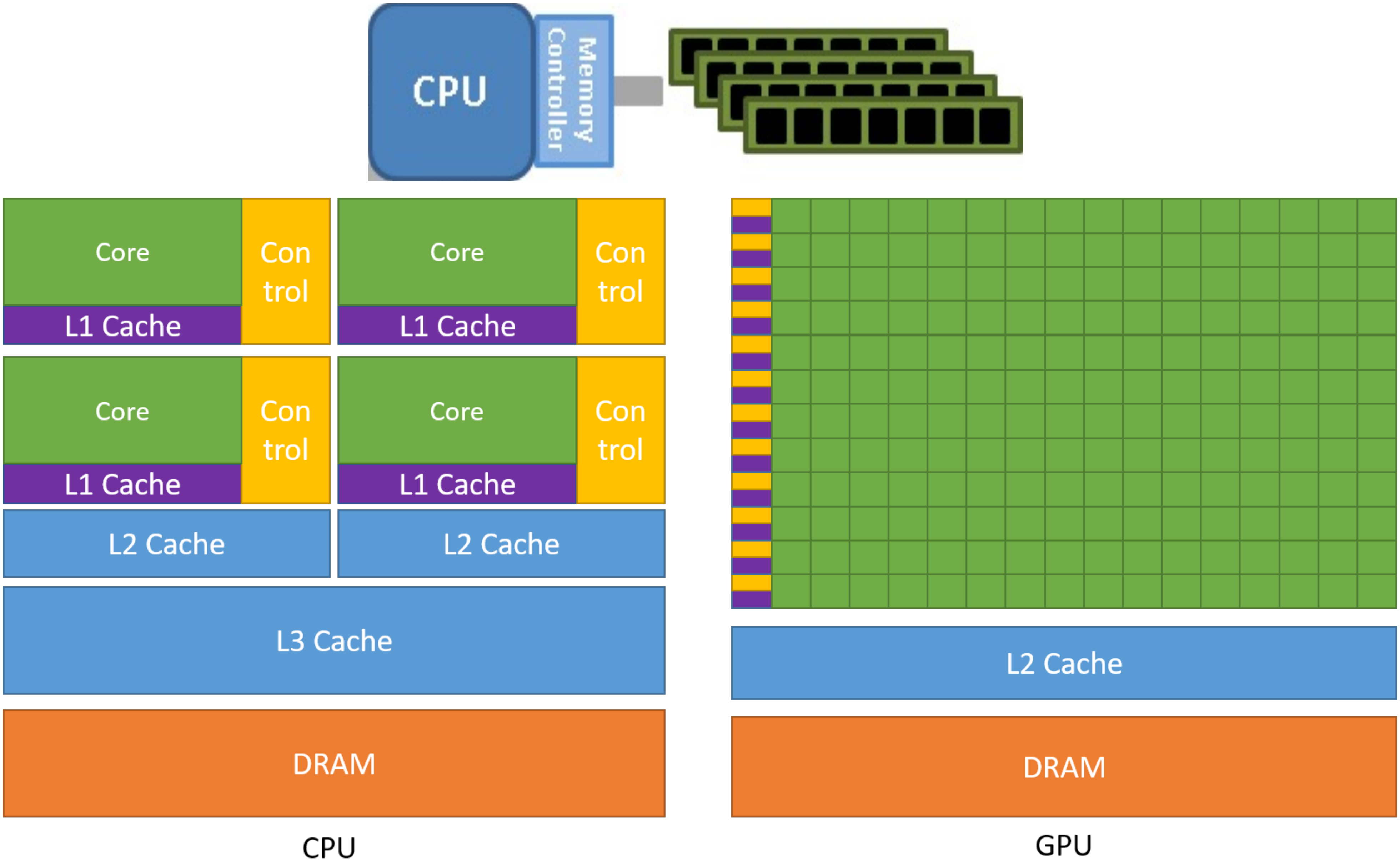
$$x \leftarrow x - \gamma_k \cdot \nabla f_{s_k}(x)$$



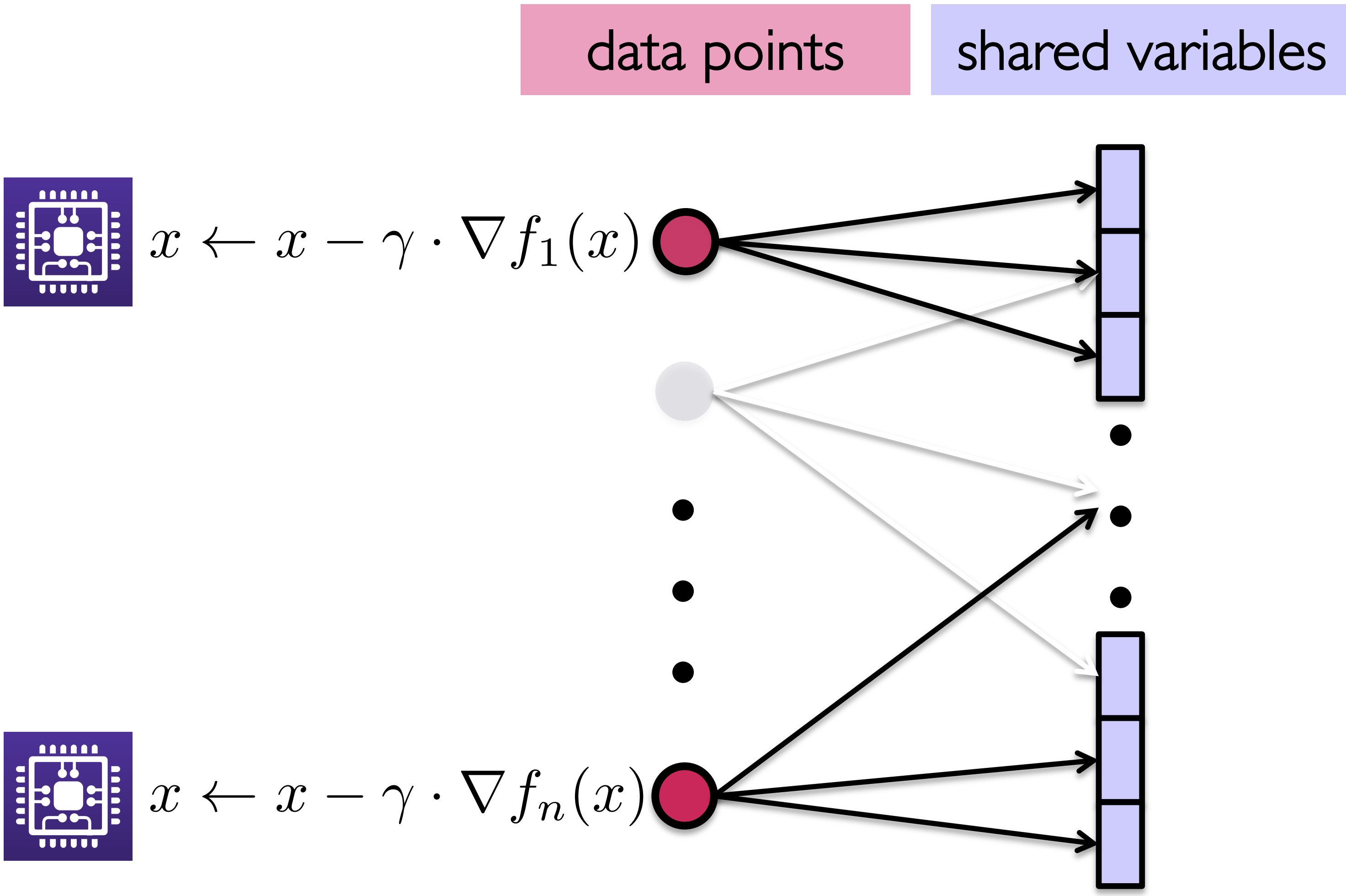
Step4:
Update Model

Parallelizing Sparse SGD on shared memory architectures

Single Machine, Multi-core

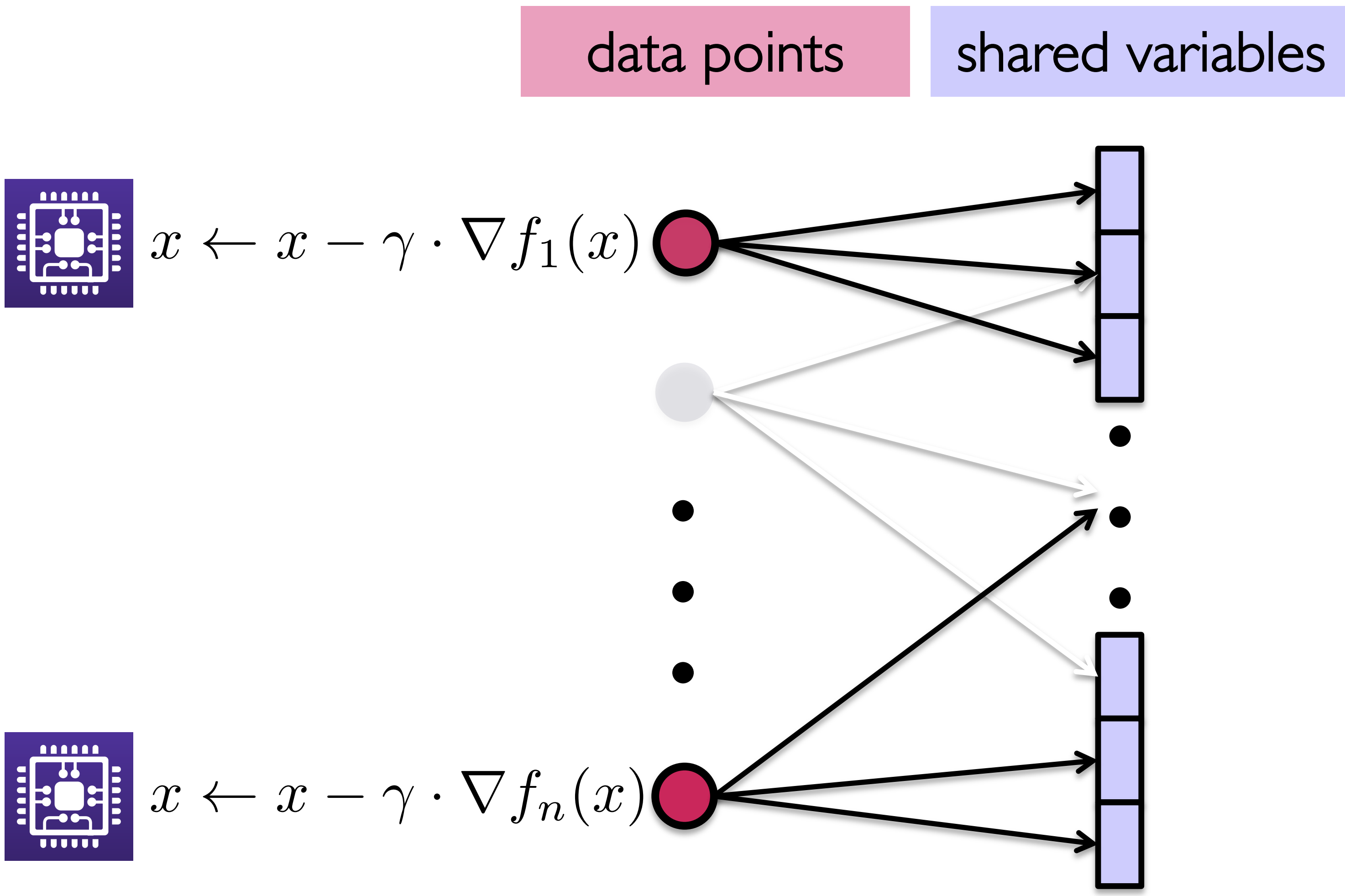


Challenges in Parallel SGD



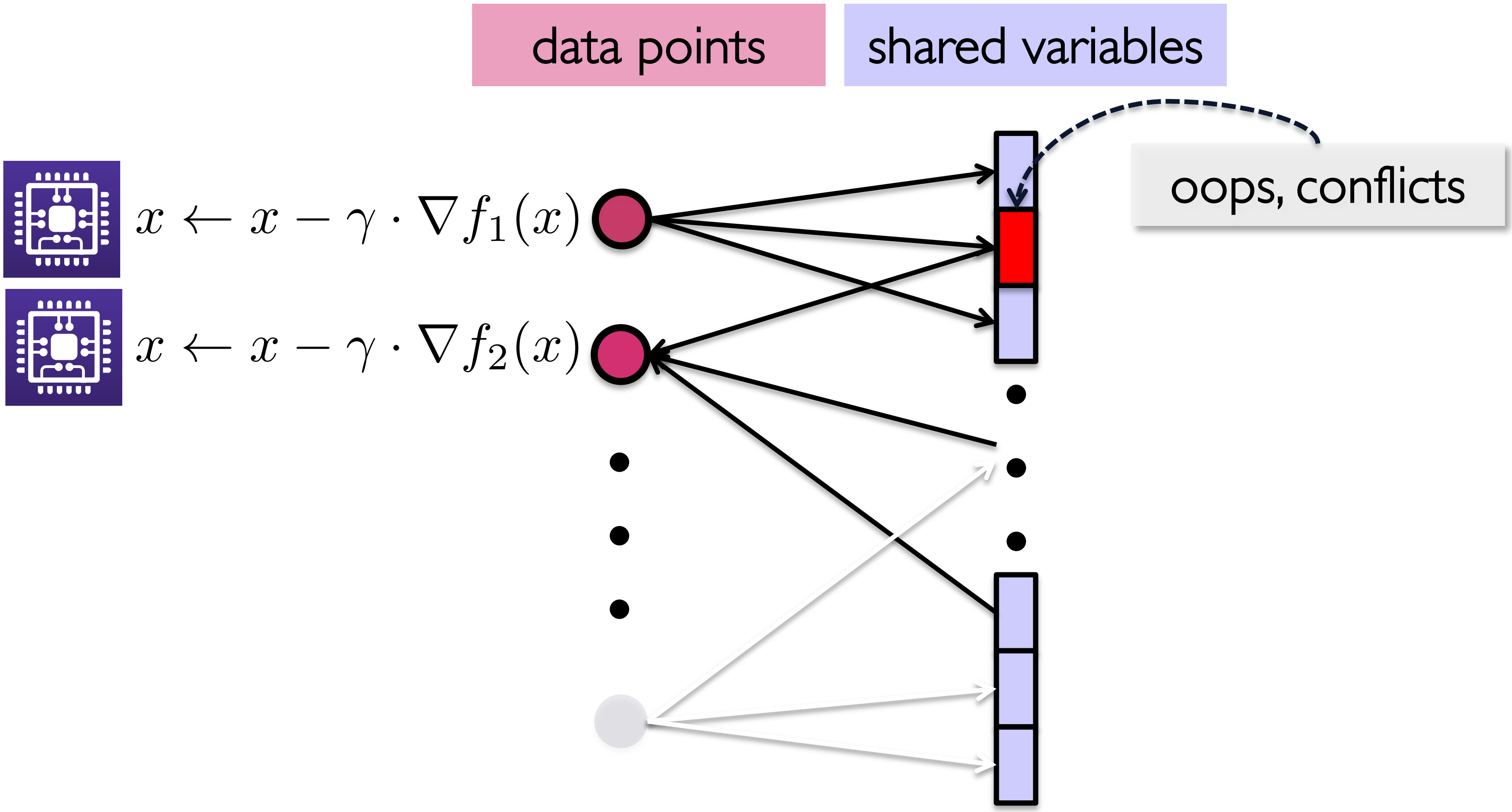
No conflict =>
2 parallel iterations = 2 serial iterations

Challenges in Parallel SGD



No conflict => Speedup

Challenges in Parallel SGD



What should we do for conflicts?

Approach 1: *Coordinate or Lock*

Approach 2: *Don't Care (Lock-free Async.)*

Prior to 2011 Work

Long line of theoretical work since the 60s
[Chazan, Miranker, 1969]

Foundational work on Asynchronous Optimization
Master/Worker model [Tsitsiklis, Bertsekas, 1986, 1989]

Recent hardware/software advances renewed the interest
Round-robin approach [Zinkevich, Langford, Smola, 2009]
Average Runs [Zinkevich et al., 2009],
Average Gradients [Duchi et al, Dekel et al. 2010]

Many based on "Coordinate" or "Lock" approach

Why Coordinate or Lock?

Issue: Synchronization and comm. overheads

HOGWILD! 2011

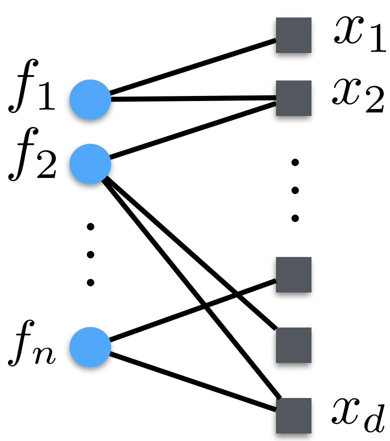
"Run parallel lock-free SGD without synchronization"


Niu


Recht


Ré


Wright



Each processor in parallel

sample function f_i

$x =$ read shared memory

$g = -\gamma \cdot \nabla f_i(x)$

for v in the support of f **do**

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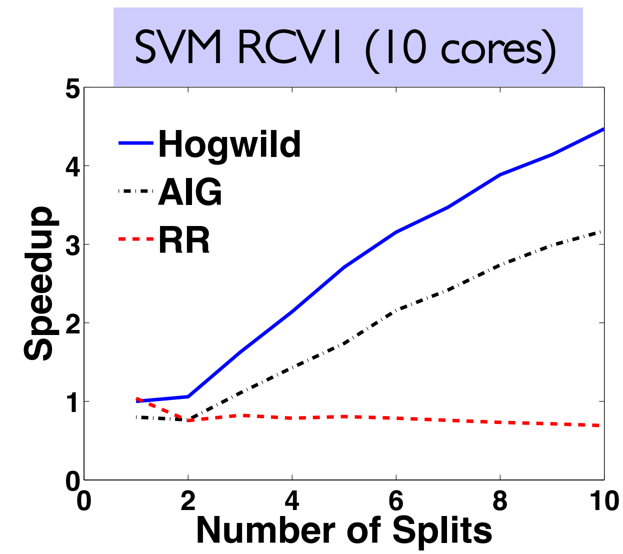


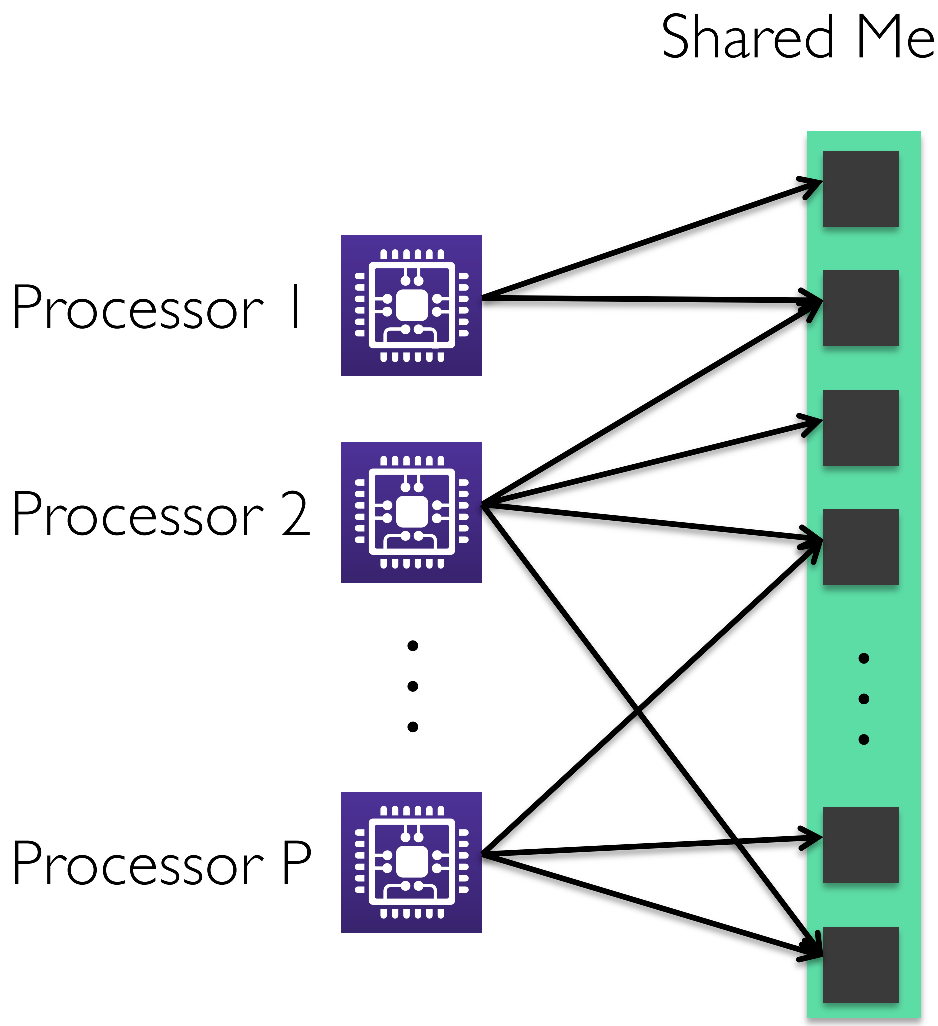
<u>Impact</u>

Google Downpour SGD, Microsoft Project Adam *use HOGWILD!*

Renewed interest on async. optimization

Challenges in Analysis

Challenges in Hogwild!



Issues:
1. updates can be old
2. results can overwritten

Incompatible with classic SGD analysis

How to Analyze Hogwild?

- Measure of performance

$$\text{worst case speedup} = \frac{\text{bound on \#iter of SGD to } \epsilon}{\text{bound on \#iter of Parallel SGD to } \epsilon}$$

Goal of a Hogwild Analysis

Prove that **Parallel SGD** and **Serial SGD** have similar convergence rates for given number of samples

Assumption:
random sampling of gradients yields a nearly optimal load balance
(if number of cores not too many)

How to Analyze Hogwild?

- [Niu, Recht, Re, and Wright, 2011] the first analysis of Hogwild! *Issues:*

  - *many impractical assumptions*
  - *simplified read/write model*
    [consistent reads, single coordinate updates, …]
  - *lengthy derivations*

- Many Async. algorithms follow using similar assumptions, and/or analysis:
  [Duchi et al, 2011], [Liu et al, 2014, 2015], [Avron et al. 2014],
  [De Sa et al, 2015], [Lian et al., 2015], [Peng et al., 2015]

How to Analyze?

[redacted]

General Framework for Asynchronous Lock-free Algorithms?

[redacted]

Analyzing Asynchronous Schemes

A Noisy Lens for Asynchronous Algorithms

Main Idea

Noisy viewpoint:

Asynchronous(*Algo.*(INPUT)) $\equiv$ Serial(*Algo.*(INPUT + Noise)

Perturbed Iterate Analysis for Asynchronous Stochastic Optimization

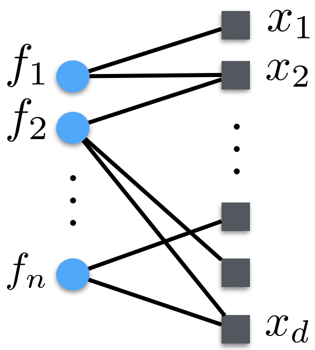
[Mania, Pan, **P**, Recht, Ramchandran, Jordan, 2015]

Joint work with





HOGWILD! as noisy SGD



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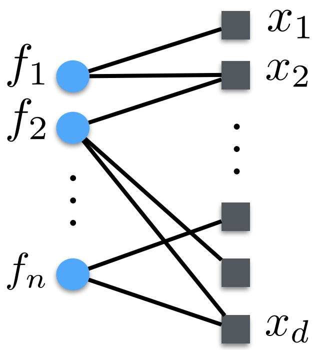
- <u>Def:</u> s_k is the k-th sampled data point

- <u>Fact:</u> Cores don't read "actual" iterates x_k but "noisy iterates" $\hat{x}_k$

- After *T* processed samples, the contents of RAM are:
  (atomic writes + commutativity)

Ex.

HOGWILD! as noisy SGD



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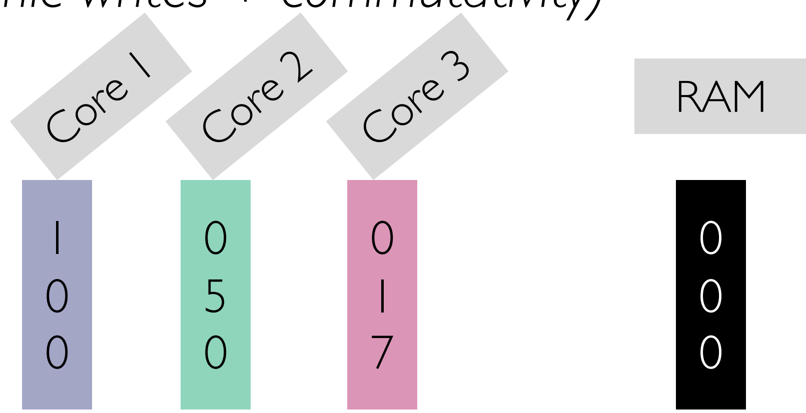

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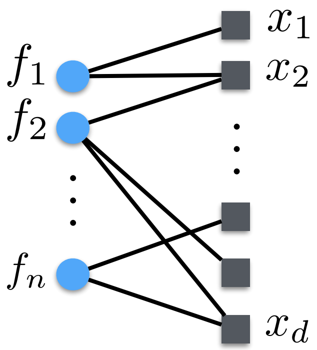

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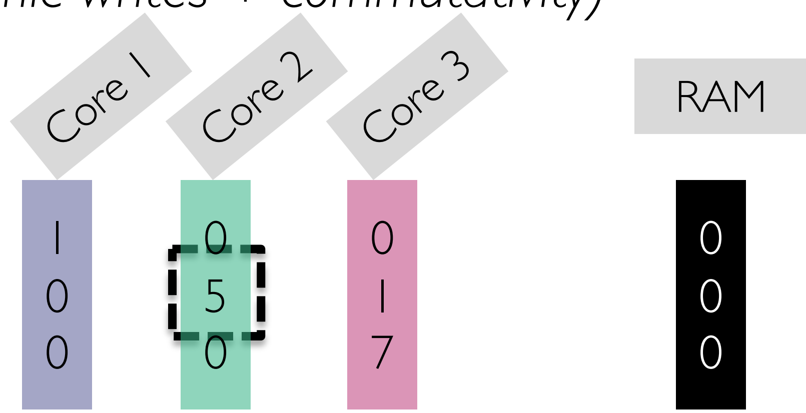

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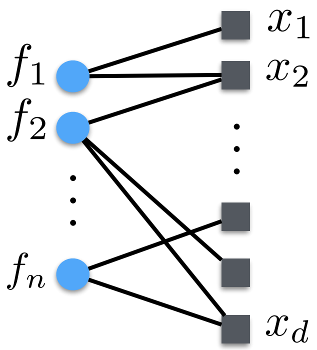

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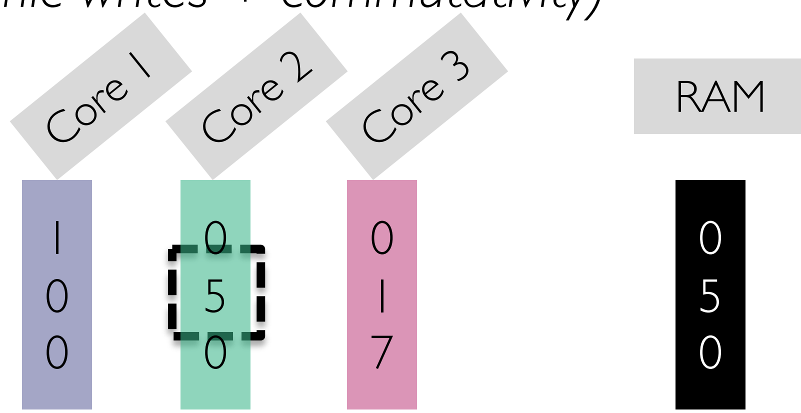

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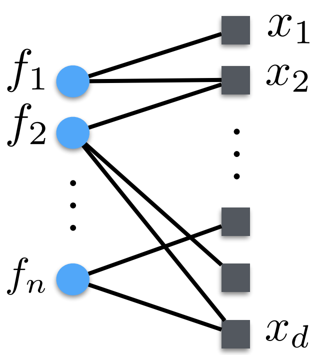

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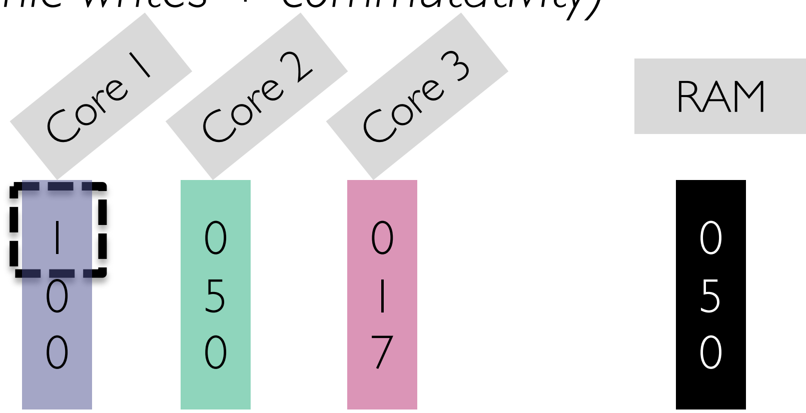

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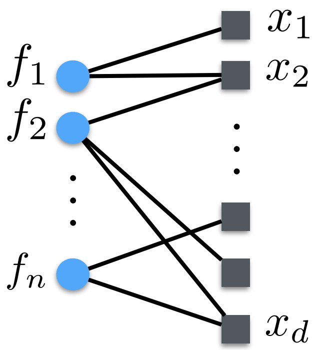

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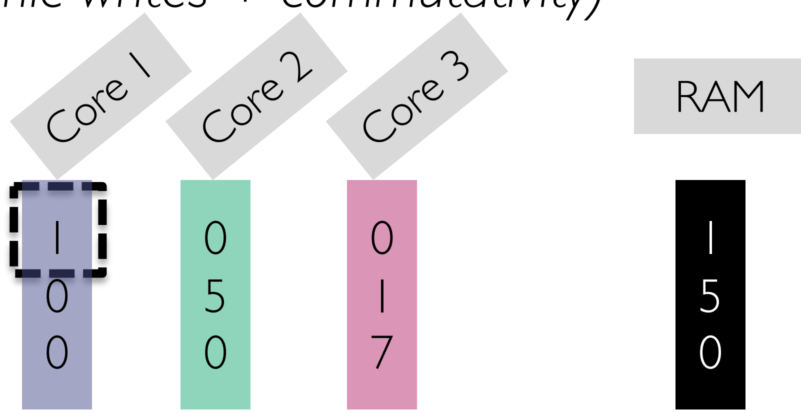

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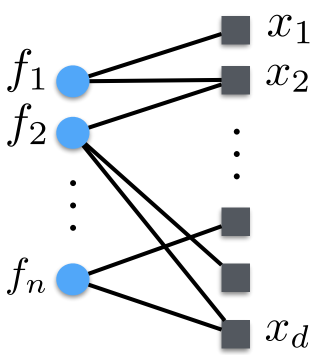

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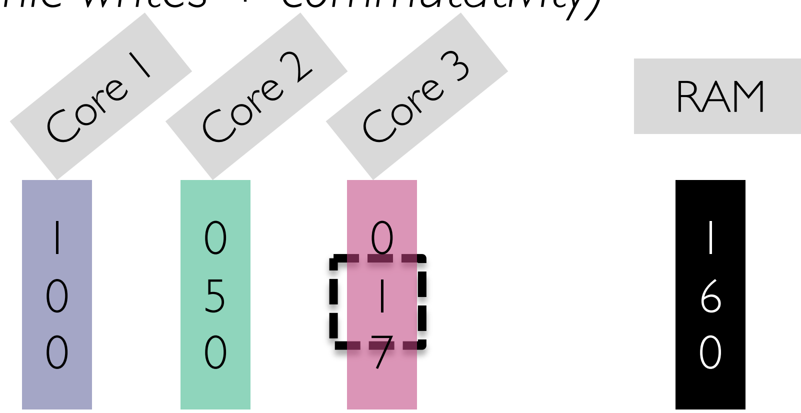

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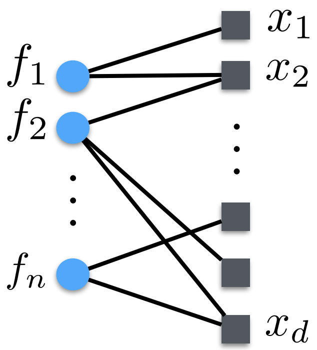

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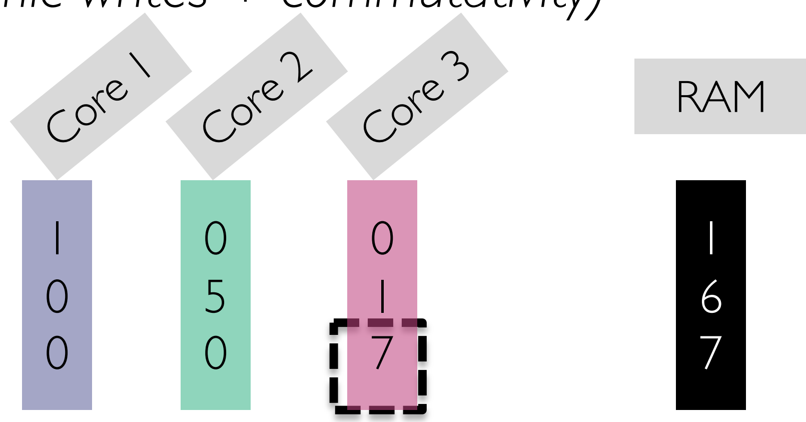

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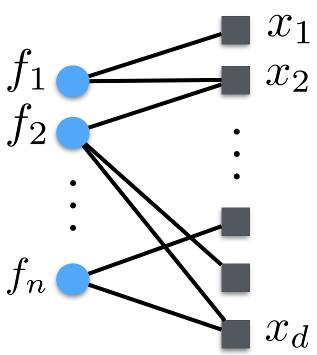

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- After *T* processed samples, the contents of RAM are: *(atomic writes + commutativity)*

$$x_0 - \gamma \cdot \nabla f_{s_0}(\hat{x}_0) - \ldots - \gamma \cdot \nabla f_{s_{T-1}}(\hat{x}_{T-1})$$

Main Questions:

1) Where does noise come from?
2) How strong is it?

Convergence Rates for Noisy SGD

We want to analyze noisy SGD

$$x_{k+1} = x_k - \gamma \cdot \nabla f_{s_k}(\hat{x}_k)$$

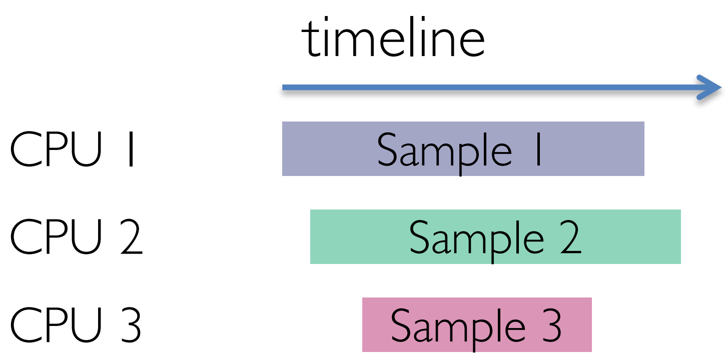
Elementary analysis (using m-strong convexity assumption on *f*):

$$\mathbb{E}\{\|x_{k+1} - x^*\|^2\} \leq (1 - \gamma \cdot m) \cdot \mathbb{E}\{\|x_k - x^*\|^2\} + \gamma^2 \cdot \mathbb{E}\{\|\nabla f_{s_k}(\hat{x}_k)\|^2\}$$

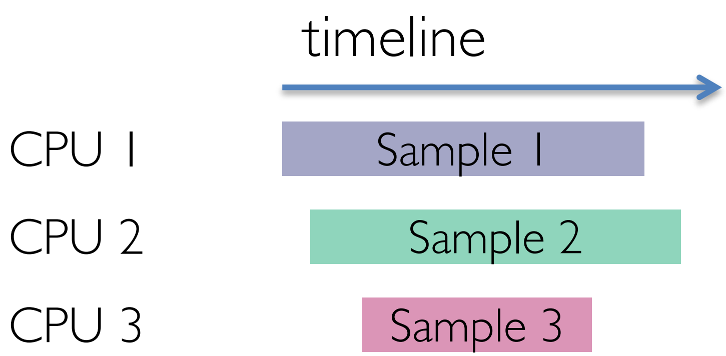
Simple Lemma:
if both terms = $O(\gamma^2 M^2)$,
Noisy SGD gets same rates as SGD *(up to multiplicative constants)*

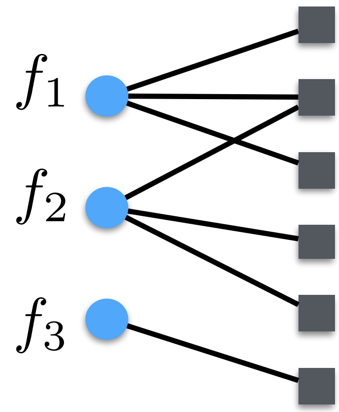
So.. is asynchrony noise small?

Understanding Asynchrony Noise

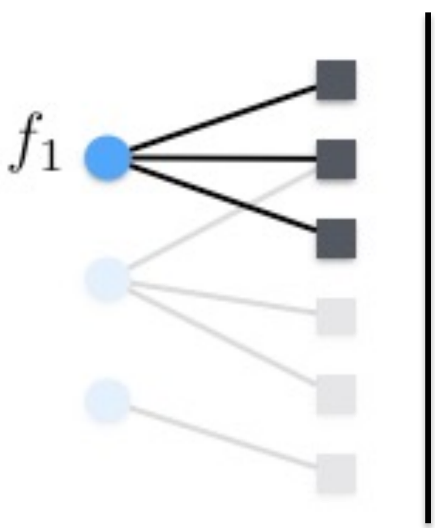


Understanding Asynchrony Noise

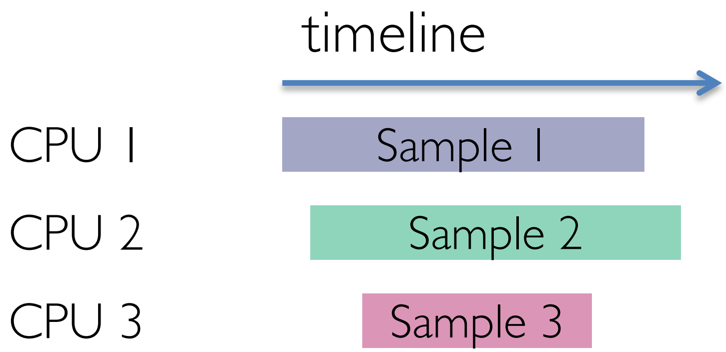


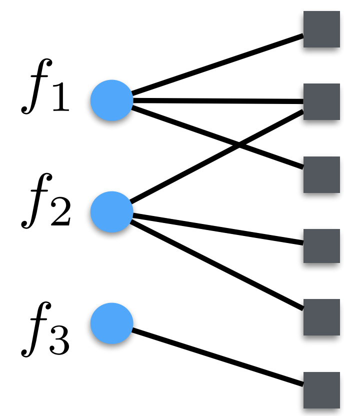


"Serialized" Processing Timeline



Understanding Asynchrony Noise





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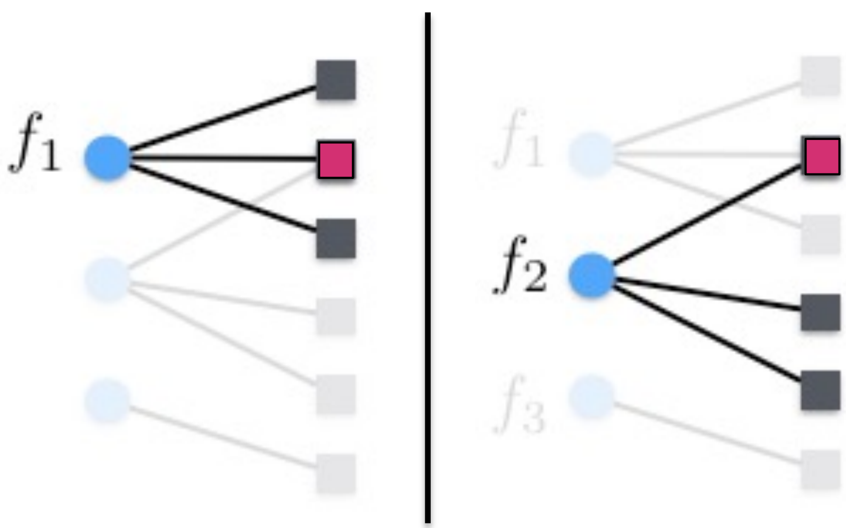
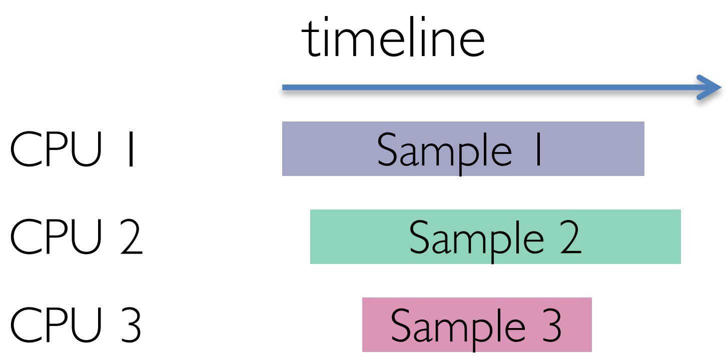
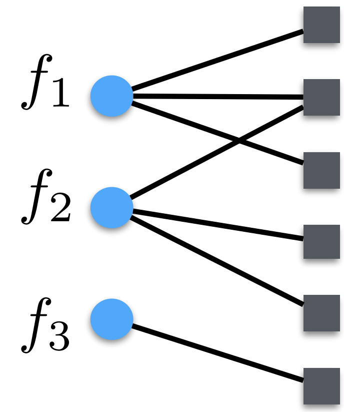
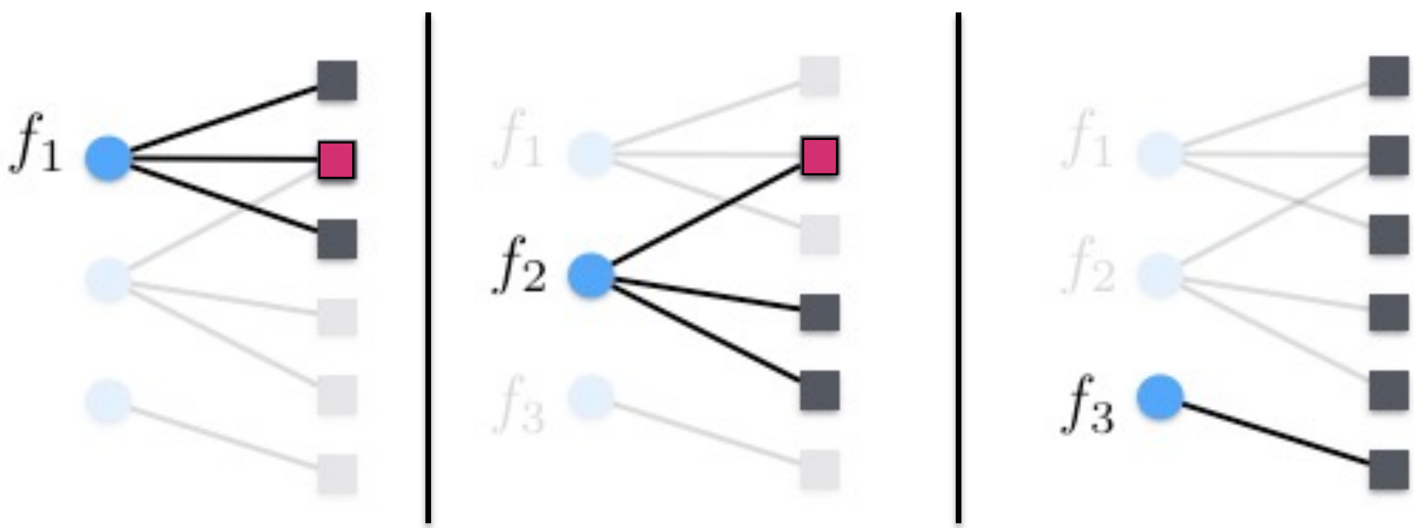


Understanding Asynchrony Noise

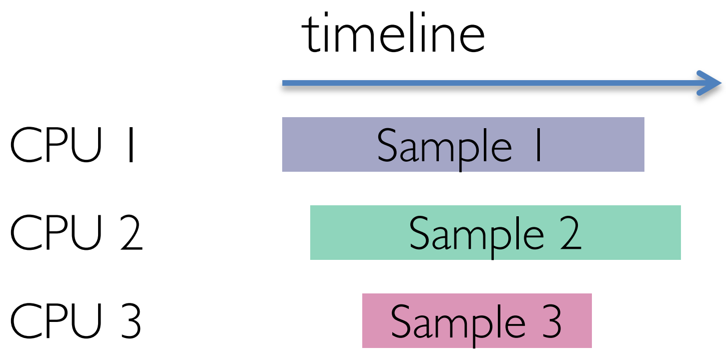


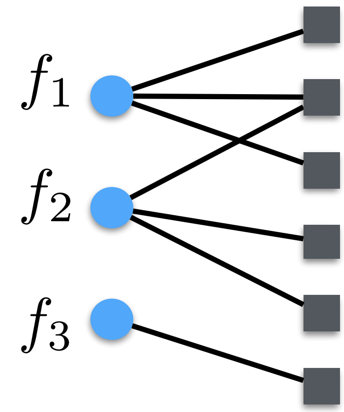


"Serialized" Processing Timeline

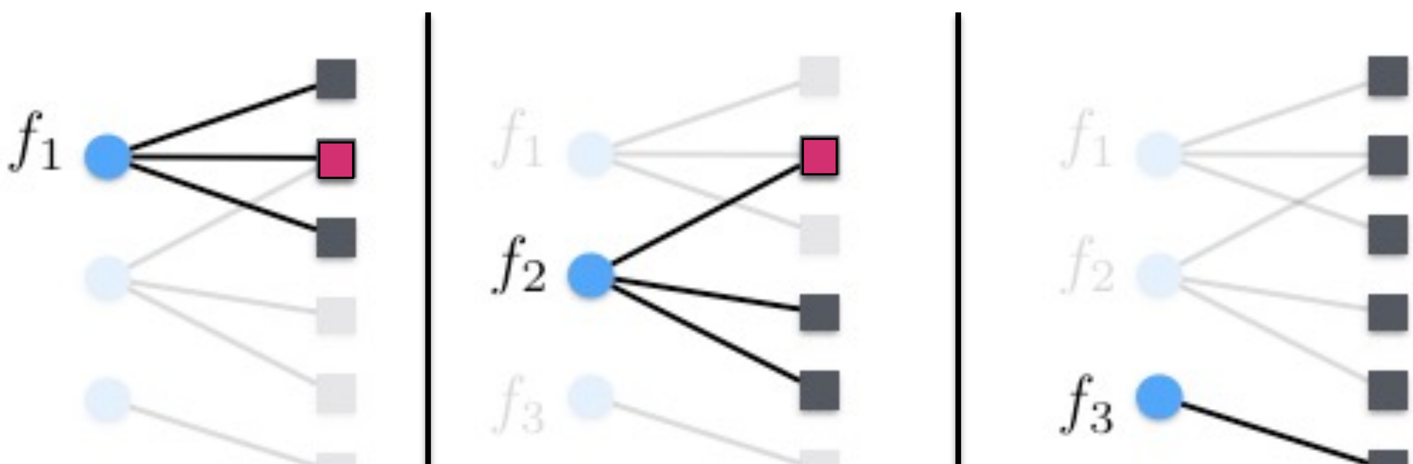


Understanding Asynchrony Noise





"Serialized" Processing Timeline



Asynchrony noise is <u>combinatorial</u>

coordinates in conflict can be as noisy as possible.

(no generative model assumptions)

Convergence Rates for Hogwild!

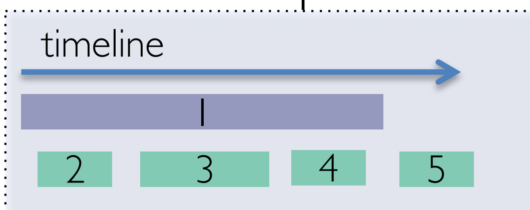
- Let's now analyze "noisy" SGD:

$$x_{k+1} = x_k - \gamma \cdot \nabla f_{s_k}(\hat{x}_k)$$

- *Assumption:* no more than τ samples processed, while a core is processing one

Eg, τ = 3

while 1 is being processed no more than 3 updates occur



Important Note:

If s_i is done before s_k is sampled:

its gradient contribution is recorded in shared RAM,

when a thread starts working on s_k

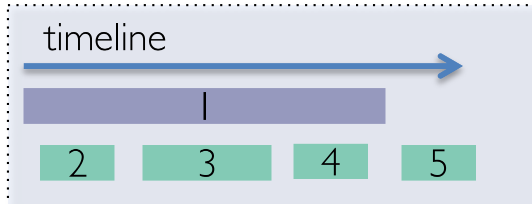
If s_i overlaps in time with s_k (i.e., the two samples are concurrently processed) :

its gradient contribution is only partially recorded in shared RAM,

when a thread starts working on s_k

Convergence Rates for Hogwild!

- *Assumption:* no more than τ samples processed, while a core is processing one



- For each sample s_k

Any difference between $\hat{x}_k$ and x_k caused only by samples that "overlap" with s_k

Therefore

- If s_i is sampled before s_k it *might* overlap with s_k iff $i \geq k - \tau$
- If s_i is sampled after s_k, it *might* overlap with s_k iff $i \leq k + \tau$

Hence:

$$\hat{x}_k - x_k = \sum_{i=k-\tau, i \neq k}^{k+\tau} \gamma \cdot S_i^j \nabla f_{s_i}(\hat{x}_i)$$

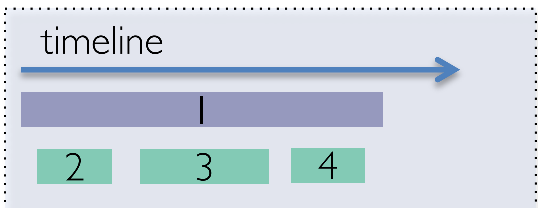
$S_i^j = \text{diagonal with entries in } \{-1, 0, 1\}$

Convergence Rates for Hogwild!

Let's now analyze "noisy" SGD:

$$x_{k+1} = x_k - \gamma \cdot \nabla f_{s_k}(\hat{x}_k)$$

<u>*Assumption:*</u> no more than τ samples processed, while a core is processing one



$$\hat{x}_k - x_k = \sum_{i=k-\tau, i \neq k}^{k+\tau} \gamma \cdot S_i^j \nabla f_{s_i}(\hat{x}_i)$$

Elementary analysis (using m-strong convexity assumption on *f*):

$$\mathbb{E}\{\|x_{k+1} - x^*\|^2\} \leq (1 - \gamma \cdot m) \cdot \mathbb{E}\{\|x_k - x^*\|^2\} + \gamma^2 \cdot \mathbb{E}\{\|\nabla f_{s_k}(\hat{x}_k)\|^2\}$$

Lemma:

if = O( )

Noisy SGD gets same rates as SGD *(up to multiplicative constants)*

Q: Is asynchrony noise that small?

Asynchrony Noise

The main thing we need to bound

$$\gamma\mathbb{E}\{\langle x_k - \hat{x}_k, \nabla f_{s_k}(\hat{x}_k)\rangle\} = \gamma^2 \mathbb{E}\left\langle \sum_{i=k-\tau, i\neq k}^{k+\tau} S_i^j \nabla f_{s_i}(\hat{x}_i), \nabla f_{s_k}(\hat{x}_k) \right\rangle$$

Reminder: we need it smaller than $\gamma^2 M^2$

Asynchrony Noise

The main thing we need to bound

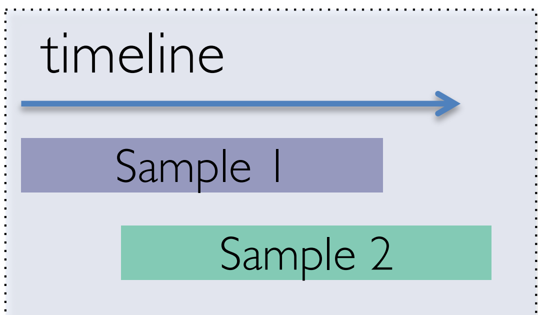
$$\boxed{\gamma\mathbb{E}\{\langle x_k - \hat{x}_k, \nabla f_{s_k}(\hat{x}_k)\rangle\}} = \gamma^2\mathbb{E}\left\langle \sum_{i=k-\tau, i\neq k}^{k+\tau} S_i^j \nabla f_{s_i}(\hat{x}_i), \nabla f_{s_k}(\hat{x}_k)\right\rangle$$

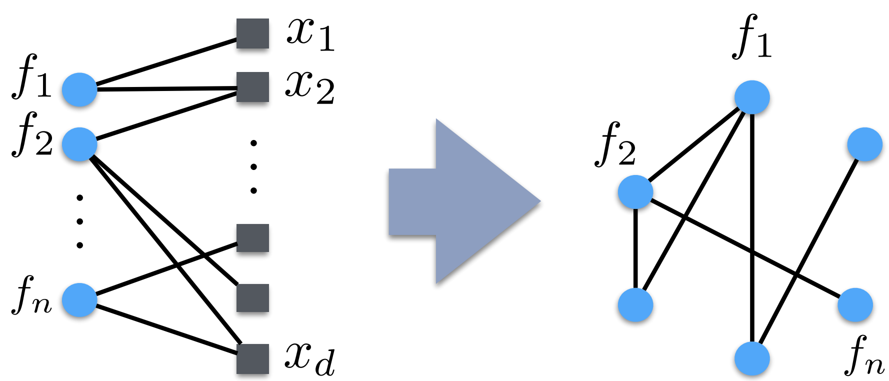
Reminder: we need it smaller than $\gamma^2 M^2$

Note: Asynchrony causes error if sampled grads overlap.

Simple Idea:

Samples might be concurrently processed, but they only "interfere" if they are talking to the same variables:





If the interference is "rare" the noise term should be small

Asynchrony Noise

The main thing we need to bound

$$\boxed{\gamma\mathbb{E}\{\langle x_k - \hat{x}_k, \nabla f_{s_k}(\hat{x}_k)\rangle\}} = \gamma^2\mathbb{E}\left\langle \sum_{i=k-\tau, i\neq k}^{k+\tau} S_i^j \nabla f_{s_i}(\hat{x}_i), \nabla f_{s_k}(\hat{x}_k) \right\rangle$$

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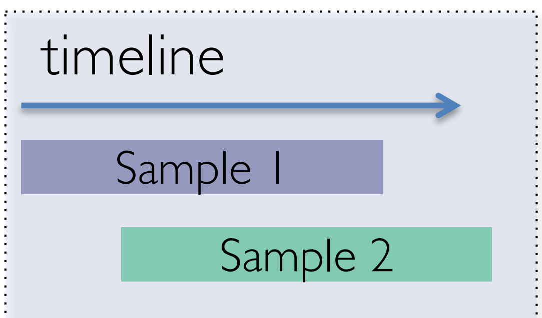

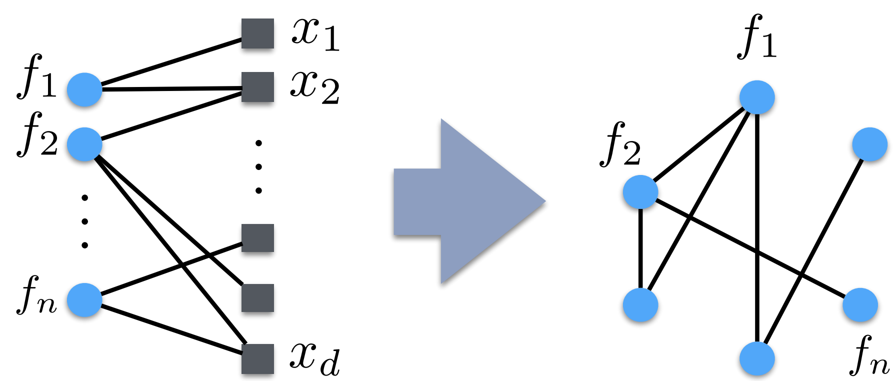

Asynchrony Noise

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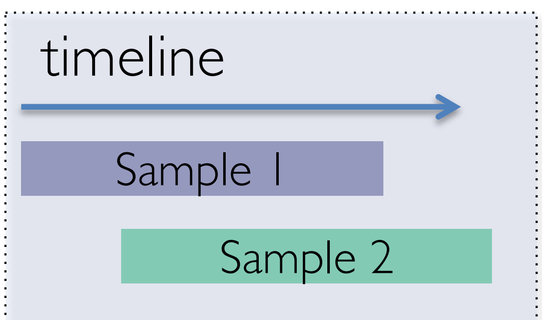
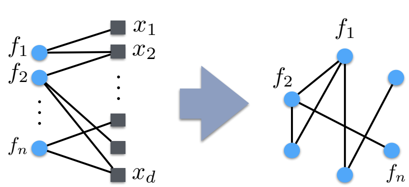
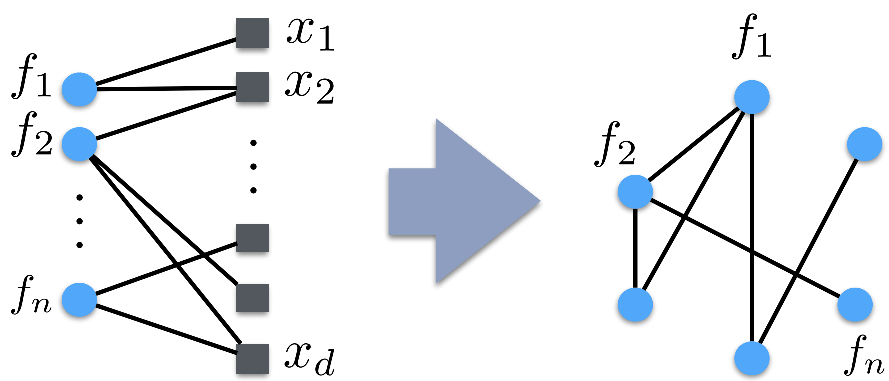






Asynchrony Noise

The main thing we need to bound

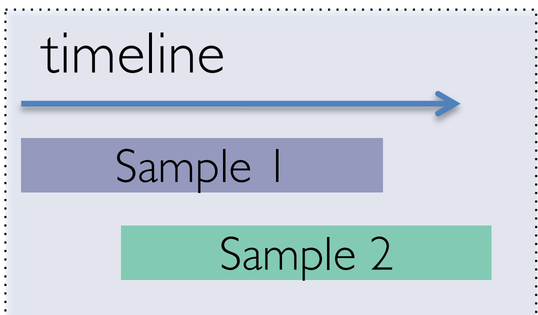
$$\boxed{\gamma\mathbb{E}\{\langle x_k - \hat{x}_k, \nabla f_{s_k}(\hat{x}_k)\rangle\}} = \gamma^2\mathbb{E}\left\langle \sum_{i=k-\tau, i\neq k}^{k+\tau} S_i^j \nabla f_{s_i}(\hat{x}_i), \nabla f_{s_k}(\hat{x}_k) \right\rangle$$

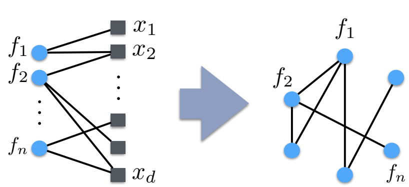
Reminder: we need it smaller than $\gamma^2 M^2$

Note: Asynchrony causes error if sampled grads overlap.

Simple Idea:

Samples might be concurrently processed, but they only "interfere" if they are talking to the same variables:





Bad Event

If the functions sampled share variables
$$\langle \nabla f_{s_i}(x_1), \nabla f_{s_j}(y_2)\rangle \neq 0$$

Good Event

If the functions sampled do not share variables
$$\langle \nabla f_{s_i}(x_1), \nabla f_{s_j}(y_2)\rangle = 0$$

Asynchrony Noise

The main thing we need to bound

$$\boxed{\gamma \mathbb{E}\{\langle x_k - \hat{x}_k, \nabla f_{s_k}(\hat{x}_k)\rangle\}} = \gamma^2 \mathbb{E} \left\langle \sum_{i=k-\tau, i\neq k}^{k+\tau} S_i^j \nabla f_{s_i}(\hat{x}_i), \nabla f_{s_k}(\hat{x}_k) \right\rangle$$

Reminder: we need it smaller than $\gamma^2 M^2$

Note: Asynchrony causes error if sampled grads overlap.

$$\gamma^2 \left\langle \sum_{i=k-\tau, i\neq k}^{k+\tau} S_i^j \nabla f_{s_i}(\hat{x}_i), \nabla f_{s_k}(\hat{x}_k) \right\rangle \leq \gamma^2 \left| \left\langle \sum_{i=k-\tau, i\neq k}^{k+\tau} \cdot S_i^j \nabla f_{s_i}(\hat{x}_i), \nabla f_{s_k}(\hat{x}_k) \right\rangle \right|$$

Cauchy-Schwarz

$$a \cdot b \leq \frac{a^2 + b^2}{2}$$

$$\|\nabla f_s(x)\|^2 \leq M^2$$

Asynchrony Noise

The main thing we need to bound

$$\boxed{\gamma \mathbb{E}\{\langle x_k - \hat{x}_k, \nabla f_{s_k}(\hat{x}_k)\rangle\}} = \gamma^2 \mathbb{E}\left\langle \sum_{i=k-\tau, i\neq k}^{k+\tau} S_i^j \nabla f_{s_i}(\hat{x}_i), \nabla f_{s_k}(\hat{x}_k) \right\rangle$$

Reminder: we need it smaller than $\gamma^2 M^2$

Note: Asynchrony causes error if sampled grads overlap.

$$\gamma^2 \mathbb{E}\left\langle \sum_{i=k-\tau, i\neq k}^{k+\tau} S_i^j \nabla f_{s_i}(\hat{x}_i), \nabla f_{s_k}(\hat{x}_k) \right\rangle \leq \gamma^2 \mathbb{E}\left\{ \sum_{i=k-\tau, i\neq k}^{k+\tau} M^2 \cdot \mathbf{1}_{s_i \cap s_k = 0} \right\}$$

What is $\mathbf{1}_{s_i \cap s_k = 0}$?

Indicator: Does sample *i* overlap with sample *k*?

Asynchrony Noise

The main thing we need to bound

$$\boxed{\gamma \mathbb{E}\{\langle x_k - \hat{x}_k, \nabla f_{s_k}(\hat{x}_k)\rangle\}} = \gamma^2 \mathbb{E}\left\langle \sum_{i=k-\tau, i\neq k}^{k+\tau} S_i^j \nabla f_{s_i}(\hat{x}_i), \nabla f_{s_k}(\hat{x}_k) \right\rangle$$

Reminder: we need it smaller than $\gamma^2 M^2$

Note: Asynchrony causes error if sampled grads overlap.

$$\gamma^2 \mathbb{E}\left\langle \sum_{i=k-\tau, i\neq k}^{k+\tau} S_i^j \nabla f_{s_i}(\hat{x}_i), \nabla f_{s_k}(\hat{x}_k) \right\rangle \leq \gamma^2 \mathbb{E}\left\{ \sum_{i=k-\tau, i\neq k}^{k+\tau} M^2 \cdot \mathbf{1}_{s_i \cap s_k = 0} \right\}$$

$$\leq \gamma^2 \cdot 2\tau \cdot M^2 \cdot \mathbb{E}\{\mathbf{1}_{s_i \cap s_k = 0}\}$$

What is $\mathbb{E}\{\mathbf{1}_{s_i \cap s_k = 0}\}$?

The probability that sample *i* overlaps with sample *k*

Asynchrony Noise

The main thing we need to bound

$$\gamma\mathbb{E}\{\langle x_k-\hat{x}_k,\nabla f_{s_k}(\hat{x}_k)\rangle\} = \gamma^2\mathbb{E}\left\langle \sum_{i=k-\tau,\, i\neq k}^{k+\tau} S_i^j \nabla f_{s_i}(\hat{x}_i), \nabla f_{s_k}(\hat{x}_k)\right\rangle$$

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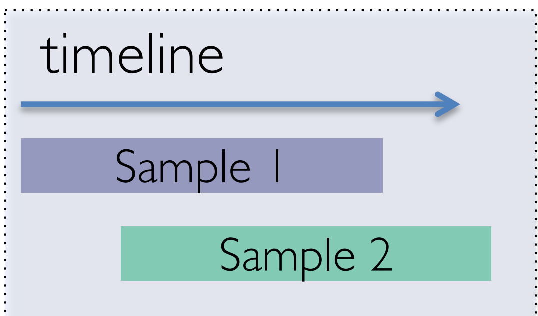
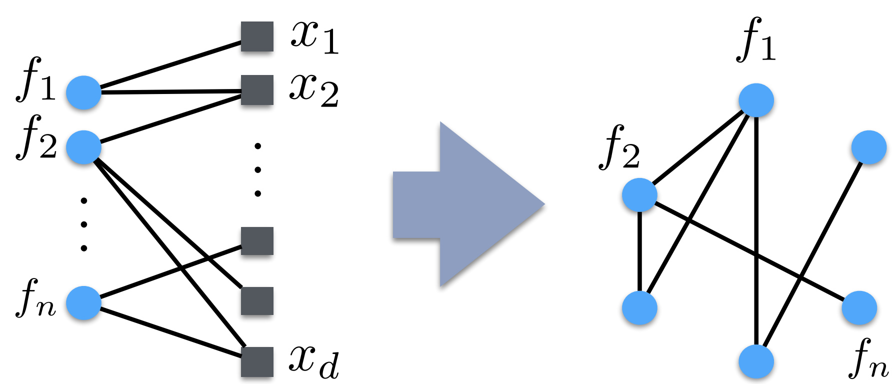
Note: Asynchrony causes error if sampled grads overlap.





Asynchrony Noise

The main thing we need to bound

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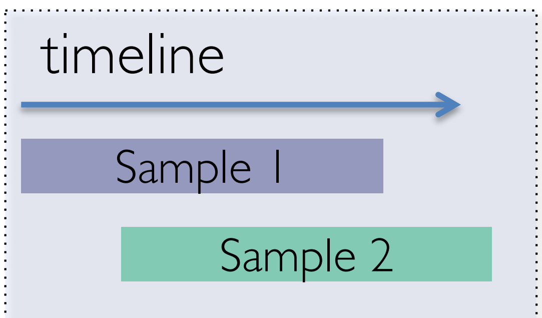
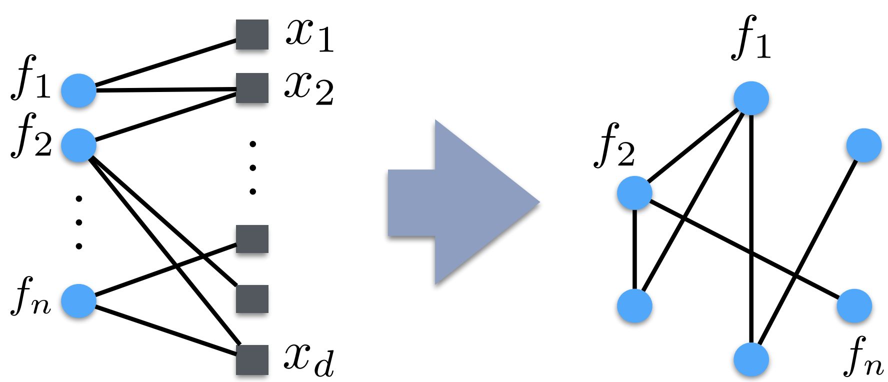
Reminder: we need it smaller than $\gamma^2 M^2$

Note: Asynchrony causes error if sampled grads overlap.





$$\boxed{\text{Pr(two samples conflict)} = \frac{\Delta_{\text{av}}}{n}}$$

Asynchrony Noise

The main thing we need to bound

$$\boxed{\gamma\mathbb{E}\{\langle x_k - \hat{x}_k, \nabla f_{s_k}(\hat{x}_k)\rangle\}} = \gamma^2\mathbb{E}\left\langle \sum_{i=k-\tau, i\neq k}^{k+\tau} S_i^j \nabla f_{s_i}(\hat{x}_i), \nabla f_{s_k}(\hat{x}_k)\right\rangle$$

Reminder: we need it smaller than $\gamma^2 M^2$

Note: Asynchrony causes error if sampled grads overlap.

$$\gamma^2\mathbb{E}\left\langle \sum_{i=k-\tau, i\neq k}^{k+\tau} S_i^j \nabla f_{s_i}(\hat{x}_i), \nabla f_{s_k}(\hat{x}_k)\right\rangle \leq \gamma^2 \cdot 2\tau \cdot M^2 \cdot \mathbb{E}\{\mathbf{1}_{s_i \cap s_k \neq 0}\}$$

The noise term is below $\gamma^2 M^2$ when $\tau \leq \dfrac{n}{2\Delta_{\text{av}}}$

Convergence Rates for Hogwild!

$$x_{k+1} = x_k - \gamma \cdot \nabla f_{s_k}(\hat{x}_k)$$

Reminder of Noisy SGD Rates:

$$\mathbb{E}\{\|x_{k+1} - x^*\|^2\} \leq (1 - \gamma \cdot m) \cdot \mathbb{E}\{\|x_k - x^*\|^2\} + \gamma^2 \cdot \mathbb{E}\{\|\nabla f_{s_k}(\hat{x}_k)\|^2\}$$
$$+ 2\gamma m \cdot \mathbb{E}\{\|x_k - \hat{x}_k\|^2\} + 2\gamma \cdot \mathbb{E}\{\langle x_k - \hat{x}_k, \nabla f_{s_k}(\hat{x}_k)\rangle\}$$

Lemma:

if = O( )

Noisy SGD gets same rates as SGD *(up to multiplicative constants)*

Hogwild Rates: Proof Recap

Hogwild is equivalent to a noisy serial SGD

asynchrony noise affects rates, but if bounded, not by much

When core delay is less than $\tau \leq \dfrac{n}{2\Delta_{\text{av}}}$, noise does not affect convergence

Hogwild! Achieves linear speedups

*=in terms of worst case convergence

Convergence of Hogwild

THEOREM 3.4. *If the number of samples that overlap in time with a single sample during the execution of* HOGWILD! *is bounded as*

$$\tau = \mathcal{O}\left(\min\left\{\frac{n}{\overline{\Delta}_C}, \frac{M^2}{\epsilon m^2}\right\}\right),$$

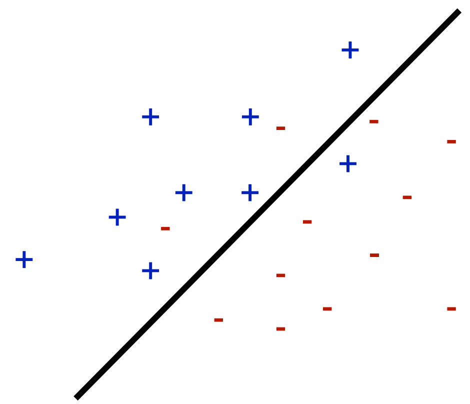
HOGWILD!, *with step size* $\gamma = \frac{\epsilon m}{2M^2}$, *reaches an accuracy of* $\mathbb{E}\|\mathbf{x}_k - \mathbf{x}^*\|^2 \leq \epsilon$ *after*

$$T \geq \mathcal{O}(1)\frac{M^2 \log\left(\frac{a_0}{\epsilon}\right)}{\epsilon m^2}$$

iterations.

Examples of Sparse Problems

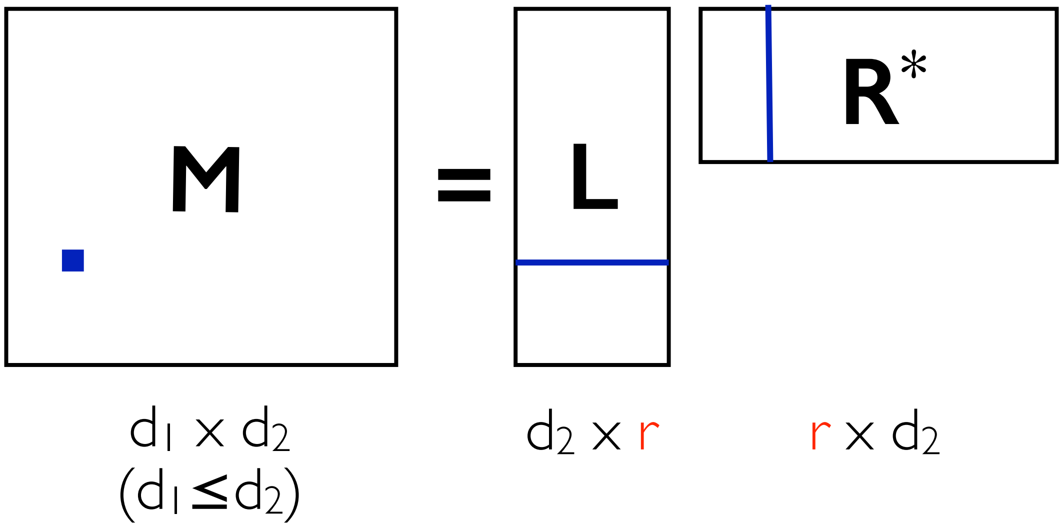
Sparse Support Vector Machines

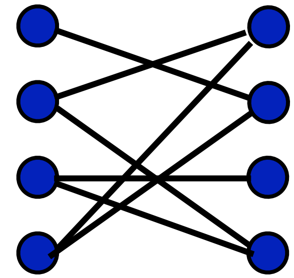


sparse vectors

$$\text{minimize}_x \sum_{\alpha \in E} \max(1 - y_\alpha x^T z_\alpha, 0) + \lambda \|x\|_2^2$$

Matrix Completion

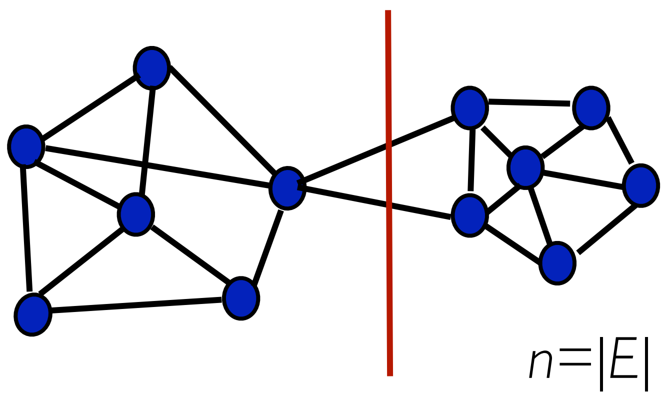




Entries Specified on set *E* (with |*E*|=*n*)

$$\mathrm{minimize}_{(\mathbf{L},\mathbf{R})} \sum_{(u,v)\in E} \left\{ (\mathbf{L}_u \mathbf{R}_v^T - M_{uv})^2 + \mu_u \|\mathbf{L}_u\|_F^2 + \mu_v \|\mathbf{R}_v\|_F^2 \right\}$$

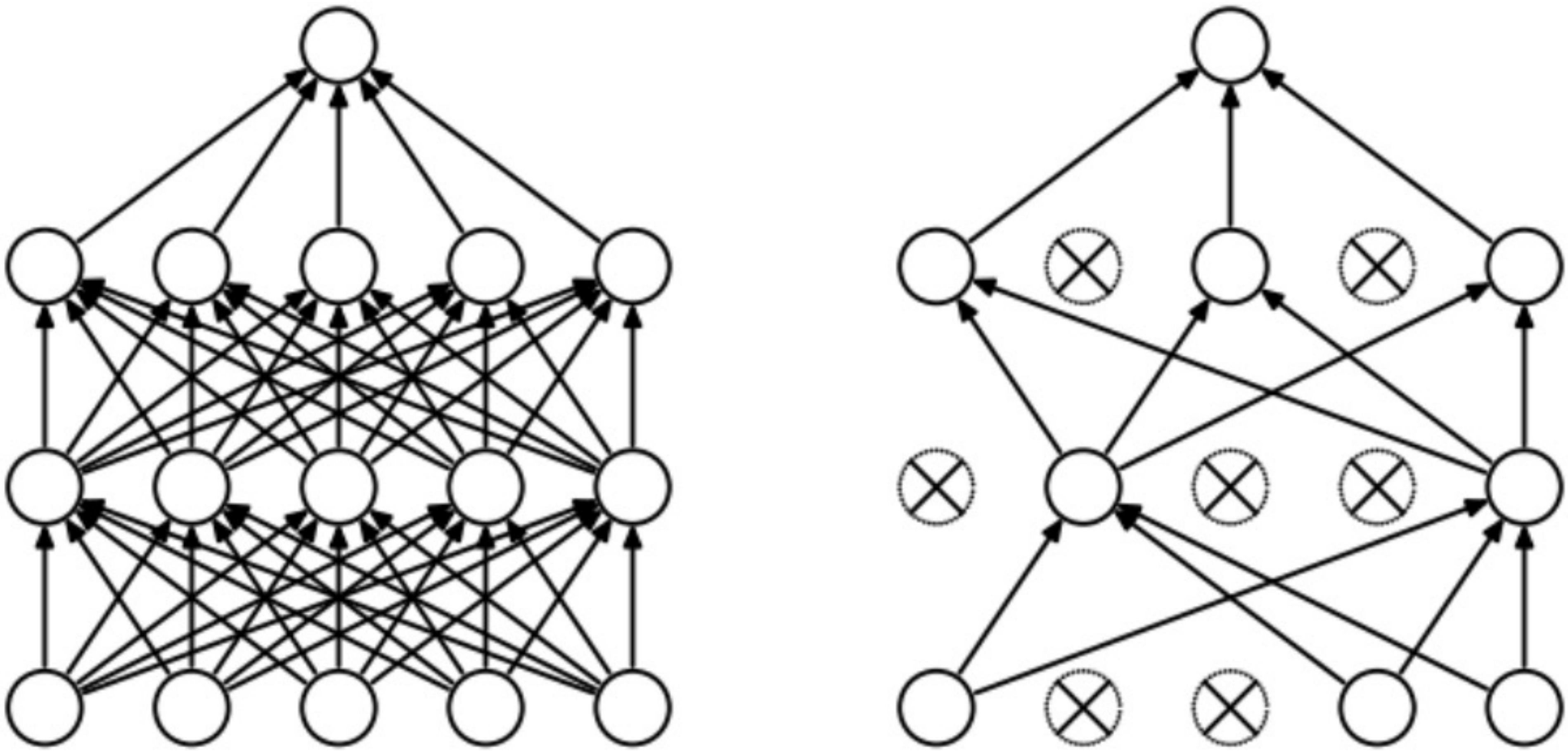
Graph Cuts



- Image Segmentation
- Entity Resolution
- Topic Modeling

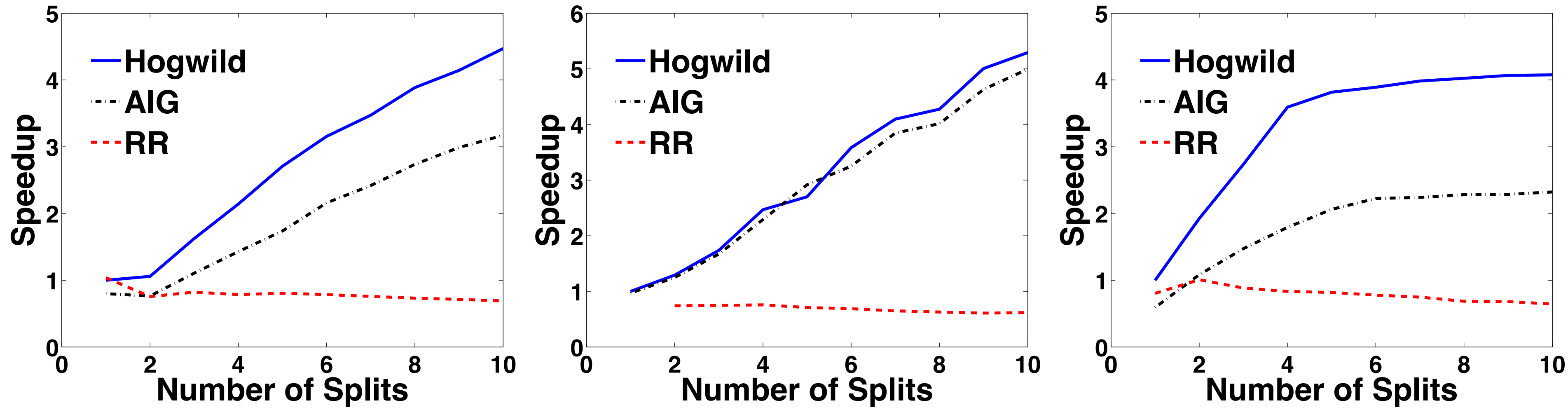
$$\begin{array}{ll} \text{minimize}_x & \sum_{(u,v)\in E} w_{uv} \|x_u - x_v\|_1 \\ \text{subject to} & \mathbf{1}_K^T x_v = 1\,,\ x_v \geq 0\,, \quad \text{for } v = 1, \ldots, D \end{array}$$

Sparsified BackProp



(a) Standard Neural Net　　(b) After applying dropout.

Speedups



SVM
RCV1

MC
Netflix

CUTS
Abdomen

Experiments run on 12 core machine

10 cores for gradients, 1 core for data shuffling

Open Problems

Open Problems: Part 1

Assumptions: Sparsity + convexity => linear speedups

O.P. :
Hogwild! On Dense Problems

Maybe we should featurize dense ML Problems,
so that updates are **sparse**

O.P. :
Fundamental Trade-off
Sparsity vs Learning Quality?

Open Problems: Part 2

- What we proved:

$$\text{worst case speedup} = \frac{\text{bound on \#iter of SGD to } \epsilon}{\text{bound on \#iter of Parallel SGD to } \epsilon}$$

- What we really care about:

$$\text{speedup} = \frac{\text{Time of serial } \mathcal{A} \text{ to accuraccy } \epsilon}{\text{Time of parallel } \mathcal{A} \text{ to accuraccy } \epsilon}$$

O.P. :
True Speedup Proofs for Hogwild

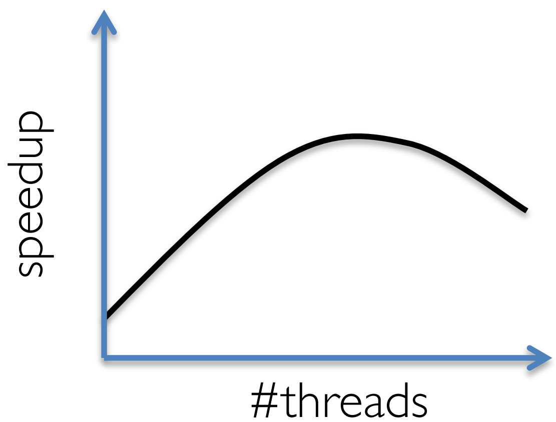
Holy Grail

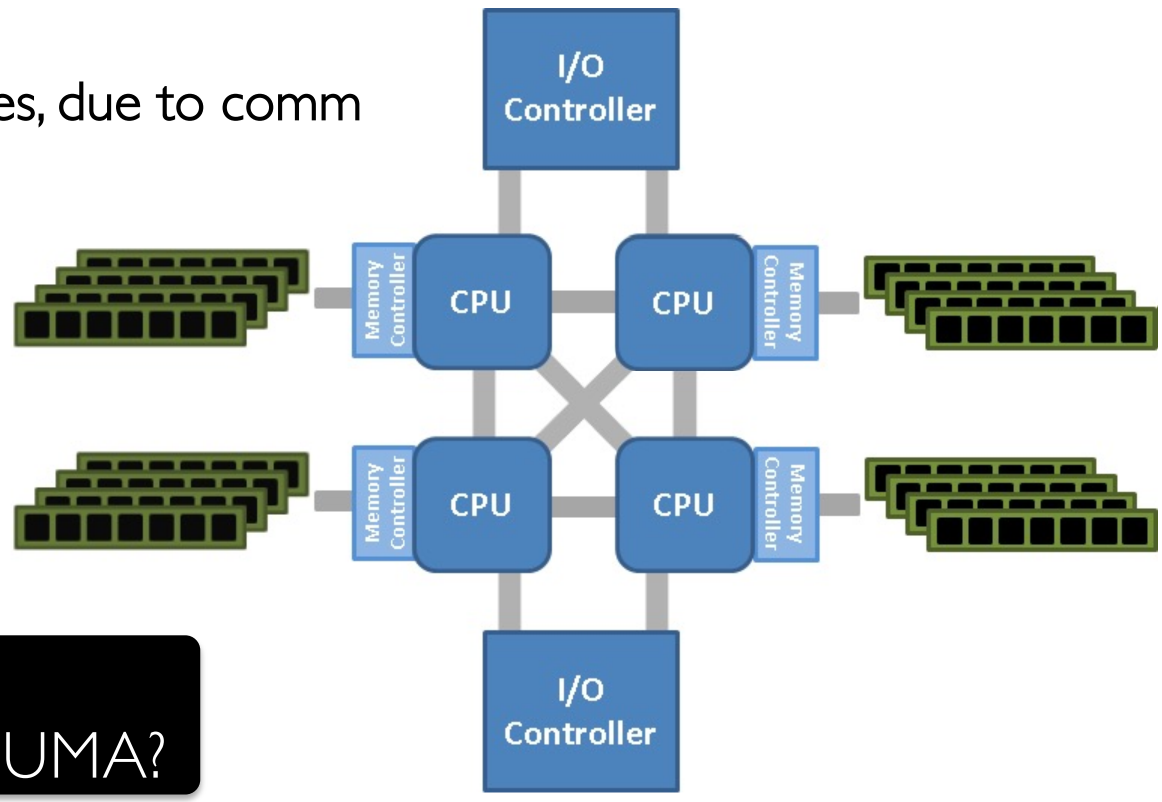
O.P. :
Guarantees for Nonconvex Problems?

Open Problems: Part 3

Hogwild! Algorithms great for Shared Memory Systems

- Issues when scaling across nodes, due to comm





O.P. :
How to provably scale on NUMA?

- Similar Issues for Distributed:

O.P. :
Sync vs Async is still open

Reproducible Models

Reproducibility

- HOGWILD! Models are not reproducible
- Each training session has inherent "system" randomness
- Does not allow to recreate models if needed
- Barrier for accountability and reproducibility
- How can we resolve it?

Reproducibility

Serial Equialence

$$A_{\text{serial}}(S, \pi) = A_{\text{parallel}}(S, \pi)$$

For all Data sets S
For all data order **π** (data points can be arbitrarily repeated)

<u>Main advantage:</u>
- we only need to "prove" speedups
- Convergence proofs inherited directly from serial

<u>Main Issue:</u>
- Serial equivalence too strict
- Cannot guarantee any speedups in the general case

Serial Equivalence

- When is serial equivalence feasible?

- What algorithmic patterns allow for efficient serial equivalent?

- Can a serial equivalent parallel algorithm ever be competitive with Hogwild?

Reading List

- Bertsekas, D.P., 1983. Distributed asynchronous computation of fixed points. Mathematical Programming, 27(1), pp.107-120. Vancouver

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- Liu, J., Wright, S., Ré, C., Bittorf, V. and Sridhar, S., 2014, June. An asynchronous parallel stochastic coordinate descent algorithm. In International Conference on Machine Learning (pp. 469-477). PMLR.

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